Name *Key* 1
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter or value for your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The value stored at the last entry of the tailTab is:

____*m*____

2. For a double hash table with $\alpha = 0.75$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

____*4*____

3. Path compression is part of which disjoint subset implementation?

____*C*____ A. Implementation 1   B. Implementation 2
C. Implementation 3   D. All three implementations

4. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Z to X, then its type will be:

____*A*____   A. Back   B. Cross   C. Forward   D. Tree

5. Which edge is chosen in a phase of Kruskal's algorithm?

____*D*____ A. The unprocessed edge (x, y) of smallest weight such that `find(x)==find(y)`
B. An edge of maximum-weight in a cycle (to be excluded)
C. An edge that is on a shortest path from the source
D. The unprocessed edge (x, y) of smallest weight such that `find(x)!=find(y)`

6. The capacity of any cut is:

____*B*____ A. A lower bound on the maximum flow.   B. An upper bound on the maximum flow.
C. The same as the capacity of all other cuts.   D. The same as the maximum attainable flow.

7. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:

____*A*____ A. finish(X) > finish(Y)   B. finish(X) < finish(Y)
C. finish(X) = finish(Y)   D. could be either A. or B.

8. Suppose an instance of bipartite matching has 4 vertices in the left column, 8 vertices in the right column, and 17 edges. The number of edges in the corresponding instance of network flow is:
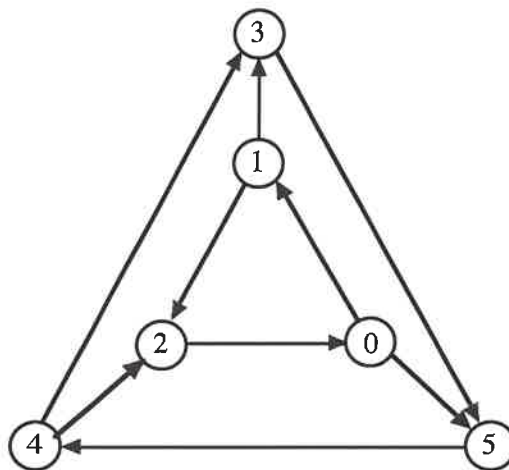
29

9. The relationship of the net flow across a cut and the amount of flow from the source to the sink is:
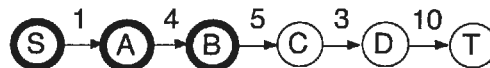
A
   A. They are equal.
   B. The amount of flow does not exceed the net flow.
   C. The net flow does not exceed the amount of flow.
   D. There is no relationship.

10. What is the number of strongly connected components in this graph?

1

11. The capacity of the following cut is _____. (S vertices are bold.)

5

12. A topological ordering of a directed graph may be computed by:

A
   A. Ordering the vertices by descending finish time after DFS
   B. Ordering the vertices by ascending discovery time after DFS
   C. Ordering the vertices by ascending finish time after DFS
   D. Ordering the vertices by descending discovery time after DFS

13. During a breadth-first search, the status of a gray vertex is:

A
   A. It is in the FIFO queue.   B. It has been completely processed.
   C. It is undiscovered.   D. It is in the priority queue.

14. What is the number of strongly connected components in this graph?
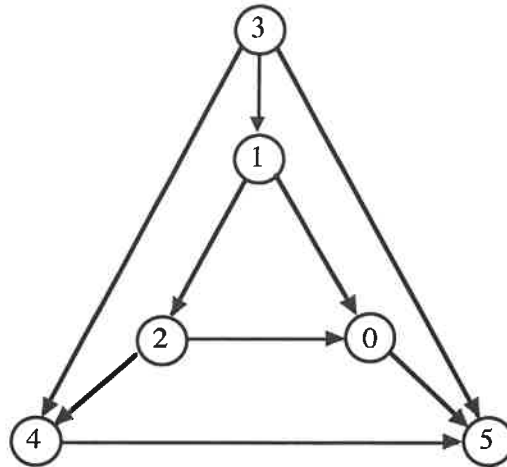
_6_

15. The worst-case time for Prim's algorithm implemented with a T-table is:

_B_ A. θ(E log V)   B. θ(V^2 + E)   C. θ(V log E)   D. θ(V log V)

16. When using two breadth-first searches to find the diameter of a tree, the purpose of the first search is to find:

_C_ A. all vertices that could be an end of a diameter.   B. both ends of a diameter.
      C. one end of a diameter.   D. the number of edges in the diameter.

17. The worst-case time for Dijkstra's algorithm implemented with a minheap is:

_B_ A. θ(V + E)   B. θ(E log V)   C. θ(V log V)   D. θ(V log E)

18. Before searching for a minimum cut in a network, it is useful to do the following:

_D_ A. Find one augmenting path.
      B. Perform a breadth-first search on the input network.
      C. Determine the type of each edge using depth-first search.
      D. Find and record augmenting paths until none remains.

19. Which person listed below has not won the Turing Award?

_B_ A. Dijkstra   B. Goldberg   C. Karp   D. Tarjan

20. What is the Edmonds-Karp variant?

_B_ A. Searching a residual network for an augmenting path of maximum capacity.
      B. Using BFS to search a residual network for an augmenting path.
      C. Using DFS to search a residual network for an augmenting path.
      D. Using the capacity of cuts to bound the amount of flow.

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.

Before

3	9(1)
4	15(2)
5	12(9)
6	8(1)
7	∞
8	∞
10	11(9)

After

3	2(6)
4	15(2)
5	4(6)
7	∞
8	∞
10	11(9)

2. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 11$ and $h_2(key) = 1 + (key \% 10)$. ***Show your work***.

```
0    220
1    660
2    442
3    333
4
5    555
6
7    777
8    882
9    999
10
```

a. Suppose 111 is to be inserted (using double hashing). Which slot will be used? (5 points)

$$P.S. = 1, 3, 5, 7, 9, 0, 2, \boxed{4}$$

$h_1 = 1$

$h_2 = 1 + 1 = 2$

b. Suppose 1001 is to be inserted (using double hashing) ***after*** 111 has been stored. Which slot will be used? (5 points)

$$P.S. = 0, 2, 4, \boxed{6}$$

$h_1 = 0$

$h_2 = 1 + 1 = 2$

3. Show the *compressed* adjacency list representation for this weighted, directed graph. (Answers using conventional adjacency lists will receive no credit.) 10 points.
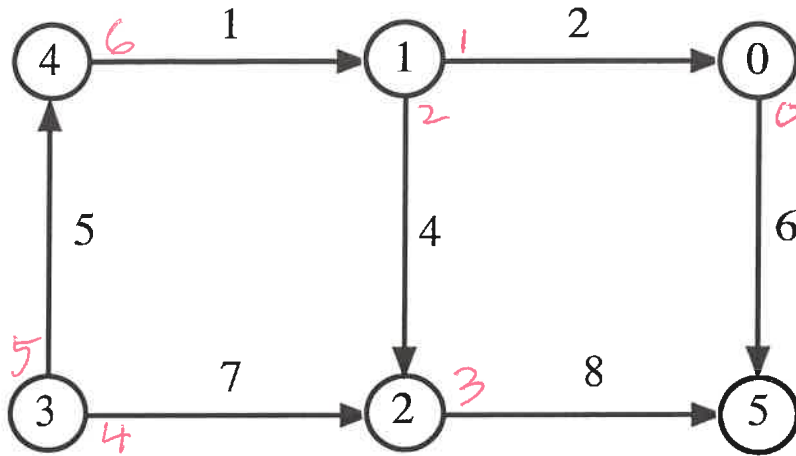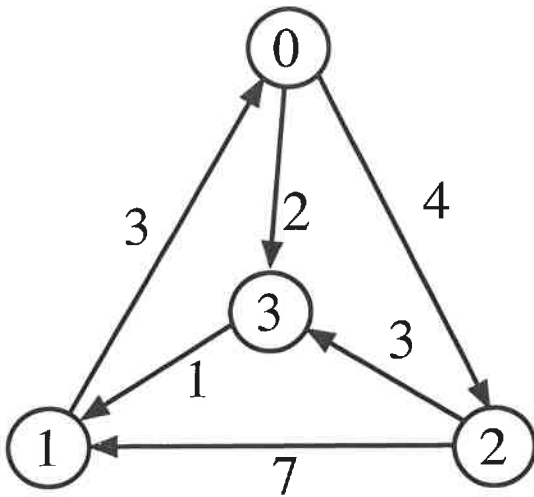

tailTab

0	0
1	1
2	3
3	4
4	6
5	7
6	7

headTab

	head	weight
0	5	6
1	0	2
2	2	4
3	5	8
4	2	7
5	4	5
6	1	1

	0		1		2		3	
0	6	3	3	3	4	2	2	3
1	3	0	6	0	7	0	5	0
2	7	3	4	3	11	3	3	3
3	4	1	1	1	8	1	6	1

4. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following graph. The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

5. Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points
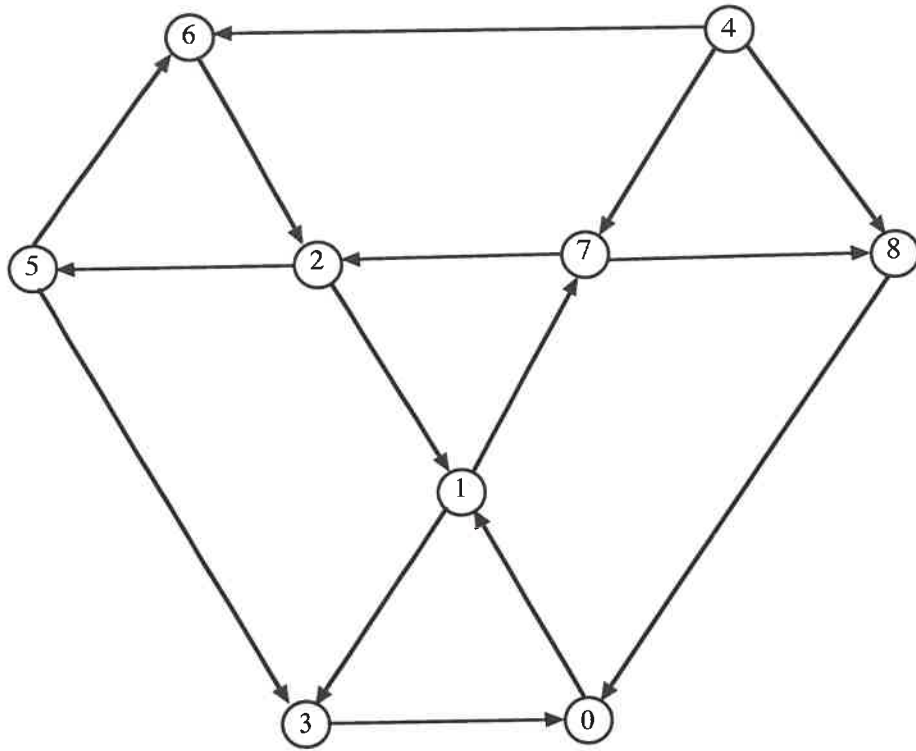
Vertex	Start	Finish	Edge	Type		Edge	Type
0	1	16	0 1	T		5 3	C
1	2	15	1 3	T		5 6	T
2	6	11	1 7	T		6 2	B
3	3	4	2 1	B		7 2	T
4	17	18	2 5	T		7 8	T
5	7	10	3 0	B		8 0	B
6	8	9	4 6	C			
7	5	14	4 7	C			
8	12	13	4 8	C			

2 errors
= 1 pt

6. Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. 0 is the source and 7 is the sink. 10 points.
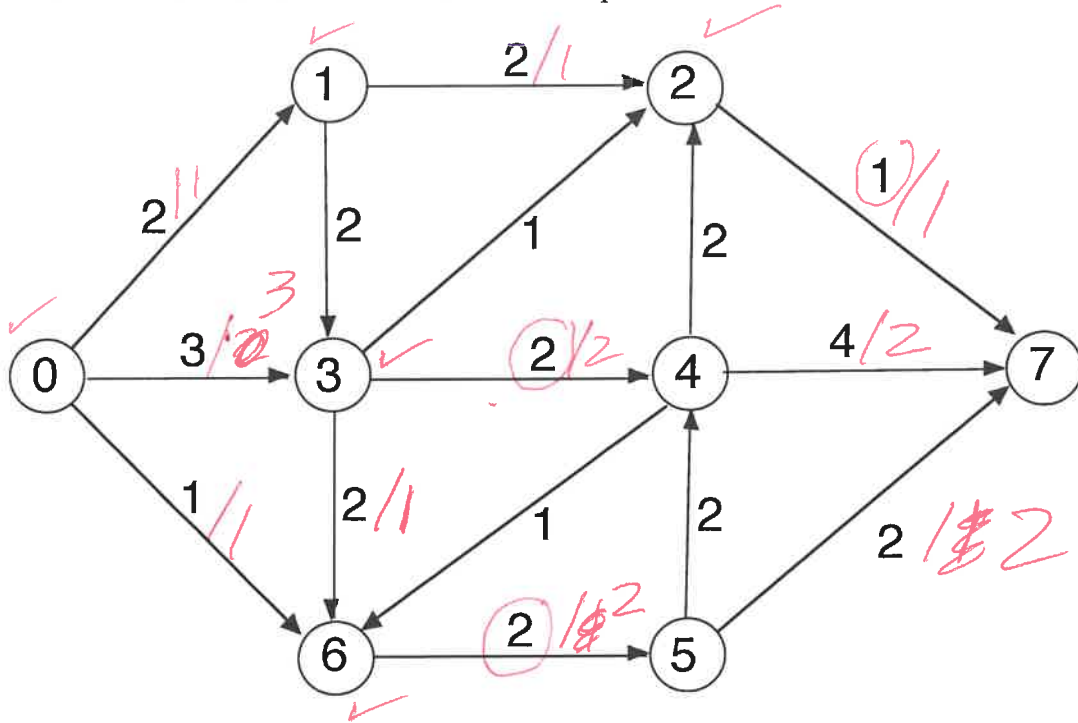
Minimum Cut:

S vertices: 0, 1, 2, 3, 6

T vertices: 7, 4, 5

Augmenting Paths and Contribution to Flow:

0, 1, 2, 7 / 1

0, 3, 4, 7 / 2

0, 6, 5, 7 / 1

0, 3, 6, 5, 7 / 1
————
5