CSE 2320
Test 1
Summer 2017

Key
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The time to run the code below is in:

```
for (i=n-1; i>=0; i--2)
  for (j=15; j<n; j+=3)
    sum+=i+j;
```

B A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

2. A sort is said to be stable when:

A A. Items with the same key will appear in the same order in the output as in the input.
B. It removes duplicate copies of any key in the final output.
C. It runs in $O(n \log n)$ time.
D. The expected time and the worst-case time are the same.

3. Which of the following is false?

C A. $n^2 \in O(n^3)$ B. $n \log n \in O(n^2)$
C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in O(g(n))$ D. $3^n \in \Omega(2^n)$

4. Bottom-up heap construction is based on applying `maxHeapify` in the following fashion:

D A. $\frac{n}{2}$ times, each time from subscript 1.
B. n - 1 times, each time from subscript 1.
C. In ascending slot number order, for each slot that is a parent.
D. In descending slot number order, for each slot that is a parent.

5. The function $2\log n + \log n$ is in which set?

D A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Omega(n \log n)$ D. $\Theta(\log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

C A. $\Omega(\log n)$ B. $\Theta(\log(n!))$ C. $O(n)$ D. $O(n^2)$

7. Which of the following is not true regarding dynamic programming?

A A. It is a form of divide-and-conquer
B. It is a form of exhaustive search
C. A cost function must be defined
D. The backtrace may be based on recomputing the cost function

8. The time to run the code below is in:

```
for (i=n-5; i>=5; i--)
  for (j=2; j<n; j=2*j+1)
    sum+=i+j;
```

A A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

9. Which sort takes worst-case $\Theta(n^2)$ time and is not stable?

D A. heap B. insertion C. merge D. selection

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A A. $T(n) \in \Omega(n^3)$ B. $T(n) \in O(\log n)$ C. $T(n) \in O(n)$ D. $T(n) \in \Omega(n^2)$

11. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 5 probes?

A A. 31 B. 63 C. 64 D. 127

12. Which of the following best approximates $H_m - H_n$? $(m > n)$

C A. H_{m-n} B. $1/(m-n)$ C. $\ln(m/n)$ D. $\ln(m-n)$

13. The function $n^2 + 3n \log n$ is in which set?

A A. $\Omega(n^2)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

14. $4^{\lg 5}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

C A. $\sqrt{7}$ B. 7 C. 25 D. 49

15. When solving the fractional knapsack problem, the items are processed in the following order.

D ______ A. Ascending order of weight
B. Ascending order of $$$/lb
C. Descending order of weight
D. Descending order of $$$/lb

16. What is the value of $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$?

D ______ A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 3

17. Suppose there is a large table with n integers, possibly with repeated values, in ascending order. How much time is needed to determine the number of occurences of a particular value?

B ______ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

18. The recursion tree for mergesort has which property?

A ______ A. each level has the same contribution
B. it leads to a definite geometric sum
C. it leads to a harmonic sum
D. it leads to an indefinite geometric sum

19. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

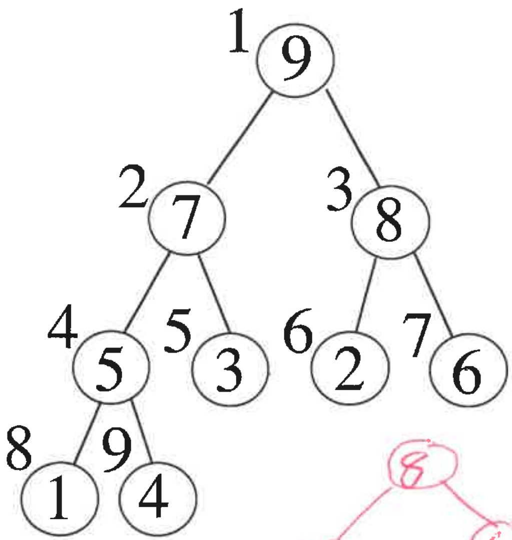
C ______ A. Ascending order of start time
B. Descending order of interval length
C. Ascending order of finish time
D. Descending order of finish time

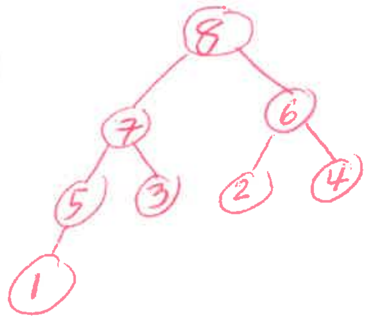
20. The goal of the Huffman coding method is:

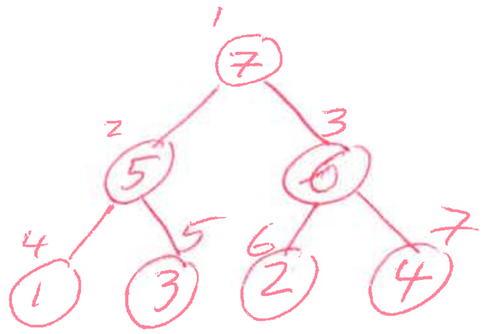
D ______ A. Construct a max-heap for the symbols in an alphabet
B. Find the symbols with high probability of occuring.
C. Maximize the compression for every string.
D. Minimize the expected bits per symbol.

Long Answer

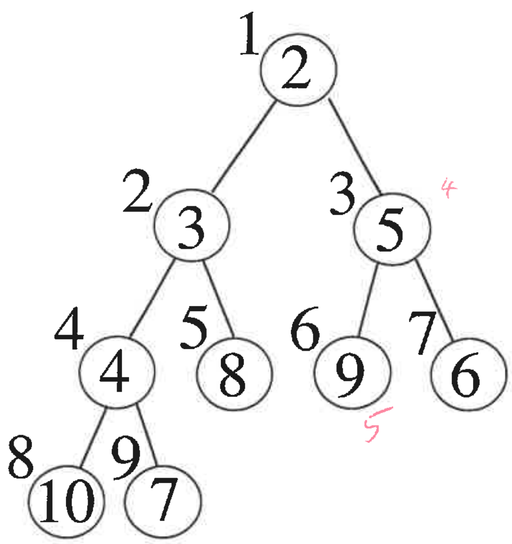
1. Show the result after performing `heapExtractMax` twice on the following maxheap. 5 points

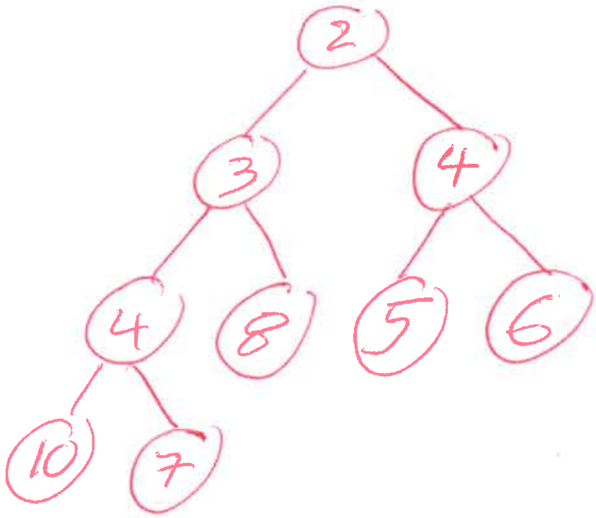




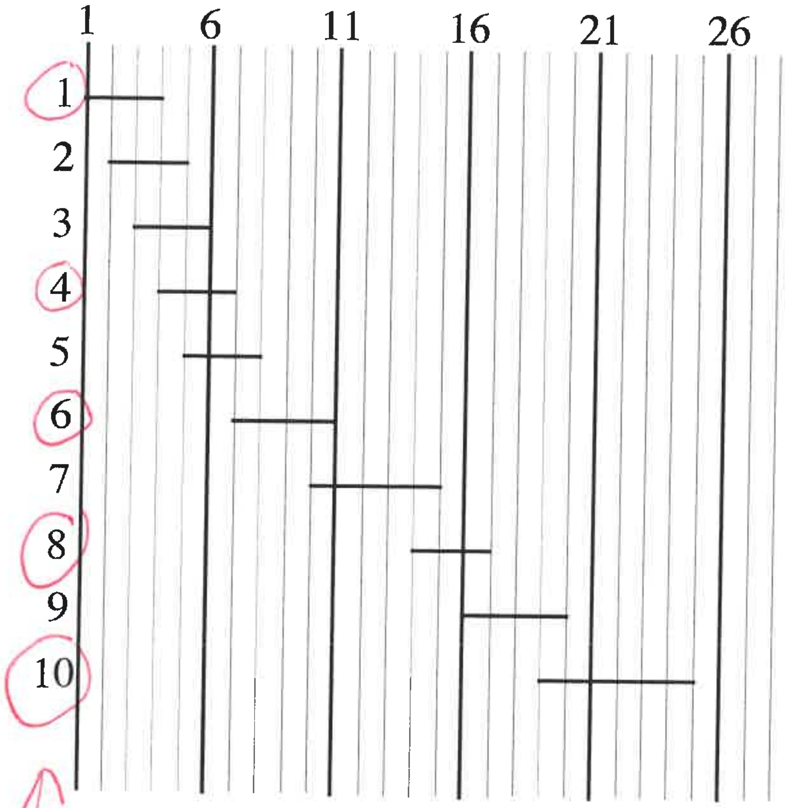


2. Show the minheap after changing (~~`max`~~ min `HeapChange`) the priority at subscript 6 to 4. 5 points





3. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



v	p	m
5	0	5
4	0	5
3	0	5
2	1	7
1	2	7
5	4	12
4	5	12
3	6	15
2	7	15
1	8	16

sum achieved

Intervals in solution {1, 4, 6, 8, 10}

4. Suppose an `int` array `a` contains `m` zeroes followed by `n` ones, where `m` and `n` are unknown non-negative values. The size of the array is given to you as `p`, i.e. `p==m+n`. Give C code to determine `m` in $O(\log p)$ time using *binary search*. (Only the code for this task, setting the value of `m`, is needed. I/O, declarations, a `return`, etc. are unnecessary. Your code must stay within the legal subscripts for array `a`.) 10 points

```
low=0;
high=p-1;
while (low<=high)
{
    mid=(low+high)/2;
    if (a[mid]==0)
        low=mid+1;
    else
        high=mid-1;
}
m=low;
```

or

m=high+1

5. Complete the following instance of the optimal matrix multiplication ordering problem, including the tree showing the optimal ordering. 10 points

p[0]=5 p[1]=4 p[2]=6 p[3]=2 p[4]=5 p[5]=6

	1		2		3		4		5	
1	0	0	120	1	88	1	138	3	???	?
2	-------		0	0	48	2	88	3	156	3
3	-------		-------		0	0	60	3	132	3
4	-------		-------		-------		0	0	60	4
5	-------		-------		-------		-------		0	0

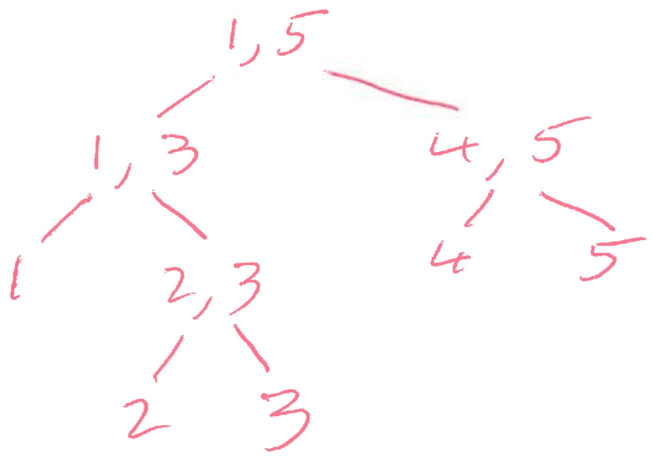
208 3

$K=1$ $\quad c(1,1)+c(2,5) + P_0 P_1 P_5$
$\quad 0 \quad 156 \quad 5 \cdot 4 \cdot 6 = 276$

$K=2$ $\quad c(1,2)+c(3,5) + P_0 P_2 P_5$
$\quad 120 \quad 132 \quad 5 \cdot 6 \cdot 6 = 432$

$K=3$ $\quad c(1,3)+c(4,5) + P_0 P_3 P_5$
$\quad 88 \quad 60 \quad 5 \cdot 2 \cdot 6 = \boxed{208}$

$K=4$ $\quad c(1,4)+c(5,5) + P_0 P_4 P_5$
$\quad 138 \quad 0 \quad 5 \cdot 5 \cdot 6 = 288$



6. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + 1$ is in $\Theta\left(\sqrt{n}\right)$. 10 points

$T(n) \Rightarrow 1$ $\qquad 1$

2

$T\left(\frac{n}{4}\right) \Rightarrow 1$ $\qquad 2$

2

$T\left(\frac{n}{16}\right) \Rightarrow 1$ $\qquad 4$

2

$T\left(\frac{n}{64}\right) \Rightarrow 1$ $\qquad 8$

$\vdots$

$T(1)$ $\qquad \#\text{leaves} = 2^{\log_4 n}$

$= n^{\log_4 2}$

$= \sqrt{n}$

$1 + \log_4 n$

$$\sqrt{n} + \sum_{k=0}^{\log_4 n - 1} 2^k = \sqrt{n} + \frac{2^{\log_4 n} - 1}{2 - 1}$$

$$= \sqrt{n} + n^{\log_4 2} - 1$$

$$= 2\sqrt{n} - 1$$

$$= \Theta(\sqrt{n})$$

7. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + 1$ is in $O\left(\sqrt{n}\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(\sqrt{n}\right)$.) 10 points

Suppose $T(k) \le c\sqrt{k}$ for $k < n$

$$T\left(\tfrac{n}{4}\right) \le c\frac{\sqrt{n}}{2}$$

$$T(n) = 2T\left(\tfrac{n}{4}\right) + 1$$

$$\le 2c\frac{\sqrt{n}}{2} + 1 = c\sqrt{n} + 1$$

Stuck

Brute expansion

$$T(1) = d$$

$$T(4) = 2d + 1$$

$$T(16) = 2(2d+1) + 1 = 4d + 3$$

$$T(64) = 2(4d+3) + 1 = 8d + 7$$

$$T(n) = d\sqrt{n} + \sqrt{n} - 1$$

$$= (d+1)\sqrt{n} - 1 = c\sqrt{n} - 1$$

Suppose $T(k) \le c\sqrt{k} - 1$ for $k < n$

$$T\left(\tfrac{n}{4}\right) \le c\frac{\sqrt{n}}{2} - 1$$

$$T(n) = 2T\left(\tfrac{n}{4}\right) + 1$$

$$\le 2\left[c\frac{\sqrt{n}}{2} - 1\right] + 1$$

$$= c\sqrt{n} - 2 + 1$$

$$= c\sqrt{n} - 1$$