

Multiple Choice:

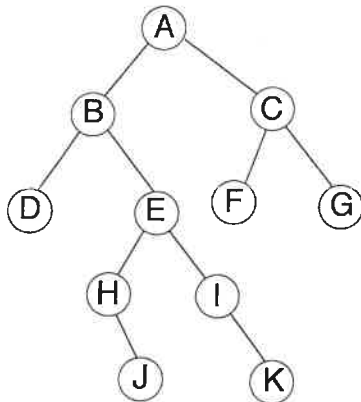
1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. In a binary search tree, which element does not have a predecessor?

C

- A. any one of the leaves B. the maximum C. the minimum D. the root

2. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 9?



- A. A
B. C
C. F
D. G

C

3. Memoization is associated with which technique?

D

- A. bottom-up dynamic programming B. PARTITION
C. greedy methods D. top-down dynamic programming

4. Circular linked lists are occasionally useful because

A

- A. some operations may be done in constant time.
B. they are an alternative to binary search trees.
C. they are useful for implementing circular queues.
D. they avoid mallocs.

5. Given a pointer to a node, the worst-case time to delete the node from a doubly-linked list with n nodes in ascending order is:

A

- A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

6. Suppose that only numbers in $1 \dots 100$ appear as keys in a binary search tree. While searching for 50, which of the following sequences of keys could not be examined?

D

- A. 100, 20, 80, 30, 50 B. 10, 30, 70, 60, 50
C. 1, 100, 20, 70, 50 D. 10, 40, 70, 30, 50

7. Which sort treats keys as several digits and uses a counting sort for each position?

D

- A. counting B. insertion C. merge D. radix

8. Which phase of counting sort actually "counts"?

B

- A. first B. second C. third D. fourth

9. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

B

- A. One node is always wasted.
B. The head points to the first element and the tail points to the last element.
C. The tail points to the first element and the head points to the last element.
D. Like a circular queue, the maximum number of items is determined at initialization.

10. Which of the following will not be true regarding the decision tree for HEAP-SORT for sorting n input values?

C

- A. Every path from the root to a leaf will have $O(n \log n)$ decisions.
B. The height of the tree is $\Omega(n \log n)$.
C. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
D. There will be $n!$ leaves.

11. Suppose a node x in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete x ?

A

- A. Find the successor of x B. Inorder traversal
C. Rotate x so it becomes a leaf D. Splice the parent of x to either child of x

12. If POP is implemented as `return stack[--SP]`, then the test for an empty stack is implemented as:

C

- A. `return stack[SP++]` B. `return SP == (-1)` C. `return SP == 0` D. `stack[SP++] = X`

13. The worst-case number of comparisons for finding the k th largest of n keys using PARTITION is in which asymptotic set?

D

- A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

14. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
    if (x==null)
        return;
    traverse(x.left);
    traverse(x.right);
    // process x here
}
```

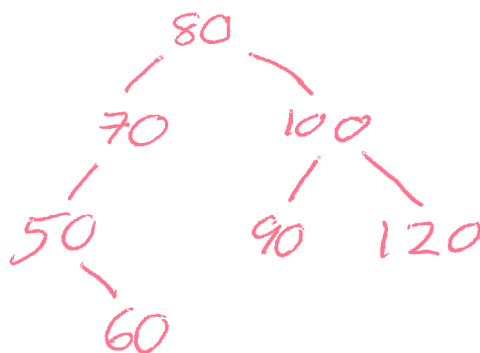
B A. inorder B. postorder C. preorder D. search for key x

15. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is in:

B A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

Long Answer

1. Give the unbalanced binary search tree that results when the keys 80, 70, 50, 100, 60, 90, 120 are inserted, in the given order, into an initially empty tree. (5 points)



2. Use dynamic programming to solve the following instance of the strictly longest increasing subsequence. Be sure to provide the table for the binary searches, along with the tables of lengths and predecessors for backtracing. (No points for solving by inspection.) (10 points)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	10	15	20	25	7	10	15	22	25	5	10	22	26	27
C	1	2	3	4	5	2	3	4	5	6	X	X	X	7 8
J	0	1	2	3	4	1	6	7	8	9	X	X	X	10 14

1: 5/1

2: ~~10/2~~ 7/6

3: ~~15/3~~ 10/7

4: ~~20/4~~ 15/8

5: ~~25/5~~ 22/9

6: 25/10

7: 26/14

8: 27/15

AB	1		9		0		7		8		2		6		3		4		5
	1	AB	9		0		7		8		2		6		3		4		5
	1	A	9		0	B	7		8		2		6		3		4		5
	1		0	A	9		7	B	8		2		6		3		4		5
	1		0	A	9		7		8	B	2		6		3		4		5
	1		0	A	9		7		8		2	B	6		3		4		5
	1		0		2	A	7		8		9	B	6		3		4		5
	1		0		2	A	7		8		9		6	B	3		4		5
	1		0		2		3	A	8		9		6		7	B	4		5
	1		0		2		3		4	A	9		6		7		8	B	5
	1		0		2		3		4	<	5	>	6		7		8		9

3. Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

1 9 0 7 8 2 6 3 4 5

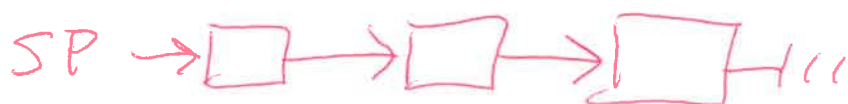
4. Provide a longest common subsequence by 1) drawing lines between instances of symbols and 2) indicating the corresponding backtrace using arrows in the matrix below. (10 points)

a b c d d c b a a b c d
a a b b c c d d a b a c

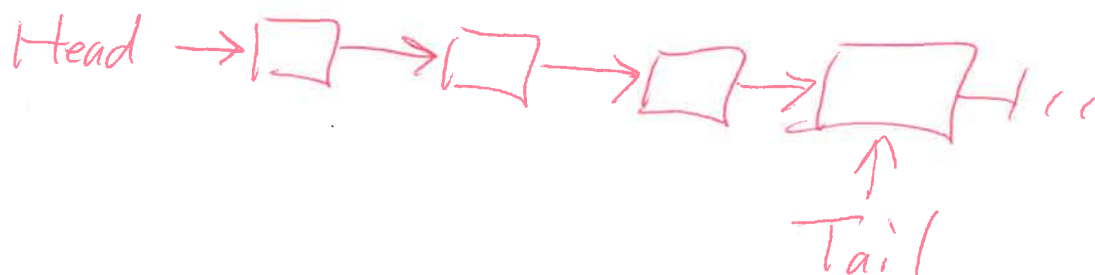
	a	a	b	b	c	c	d	d	a	b	a	c
0	0	0	0	0	0	0	0	0	0	0	0	0
a	0	1	1	1	1	1	1	1	1	1	1	1
b	0	1	1	2	2	2	2	2	2	2	2	2
c	0	1	1	2	2	3	3	3	3	3	3	3
d	0	1	1	2	2	3	3	4	4	4	4	4
d	0	1	1	2	2	3	3	4	5	5	5	5
c	0	1	1	2	2	3	4	4	5	5	5	6
b	0	1	1	2	3	3	4	4	5	5	6	6
a	0	1	2	2	3	3	4	4	5	6	6	7
a	0	1	2	2	3	3	4	4	5	6	6	7
b	0	1	2	3	3	3	4	4	5	6	7	7
c	0	1	2	3	3	4	4	4	5	6	7	8
d	0	1	2	3	3	4	4	5	5	6	7	8

5. Give diagrams showing how singly-linked lists may be used to implement the stack and queue abstractions. (10 points)

Stack



Queue



6. A billion integers in the range $0 \dots 2^{32} - 1$ will be sorted by LSD radix sort. How much faster is this done using radix $0 \dots 2^8 - 1$ rather than $0 \dots 2^4 - 1$? Show your work. (10 points)

$0 \dots 2^8 - 1$	$0 \dots 2^4 - 1$
$K = 256$	$K = 16$
$d = 4$	$d = 8$
$\Theta(d(K + n))$	$\Theta(8(16 + 1B))$
$\Theta(4(256 + 1B))$	
\uparrow	

Twice as fast