CSE 2320
Test 3
Summer 2017

Name Key
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

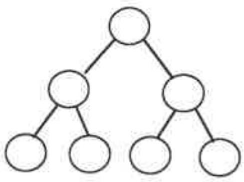
1. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The last subscript for the tailTab is:

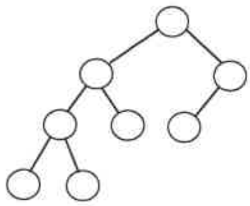
A A. n B. $n+1$ C. m D. $m+1$

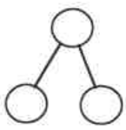
2. The expected number of probes for a successful search in hashing by chaining with α as the load factor is:

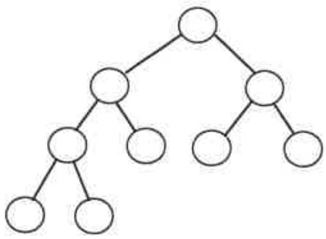
A A. α B. 2α C. $\frac{\alpha}{2}$ D. $\frac{2}{3}\alpha$

3. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?

B A. 

B. 

C. 

D. 

4. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex X to vertex Y and a path of tree edges from vertex X to Z. If there is also an edge from Y to Z, then its type will be:

B A. Back B. Cross C. Forward D. Tree

5. Which of the following cannot occur when additional edges are included in a directed graph?

D A. The graph acquires a cycle.
B. The number of strong components may remain the same.
C. The number of strong components may decrease.
D. The number of strong components may increase.

6. For a double hash table with $\alpha = 0.9$ (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

D ___ A. 1.2 B. 2 C. 5 D. 10

7. What is required when calling `union(i,j)` for maintaining disjoint subsets?

C ___ A. `i` and `j` are leaders for the same subset
B. `i` and `j` are in the same subset
C. `i` and `j` are leaders for different subsets
D. `i` is the ancestor of `j` in one of the trees

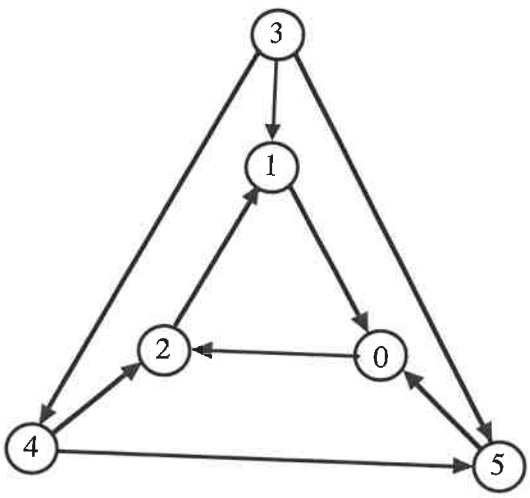
8. Suppose a directed graph has a path from vertex X to vertex Y, but no path from vertex Y to vertex X. The relationship between the finish times for depth-first search is:

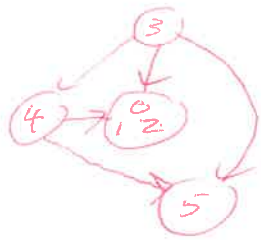
A ___ A. finish(X) > finish(Y)
B. finish(X) < finish(Y)
C. finish(X) = finish(Y)
D. could be either A. or B.

9. The cycle property for minimum spanning trees may be used to find an MST by:

C ___ A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
C. Remove the maximum weight edge in any cycle until only a tree of edges remains.
D. Remove the minimum weight edge in any cycle until only a tree of edges remains.

10. What is the number of strongly connected components in this graph?





D ___ A. 1 B. 2 C. 3 D. 4

11. Which algorithm maintains multiple subtrees?

B ___ A. Dijkstra's B. Kruskal's C. Prim's D. Warshall's

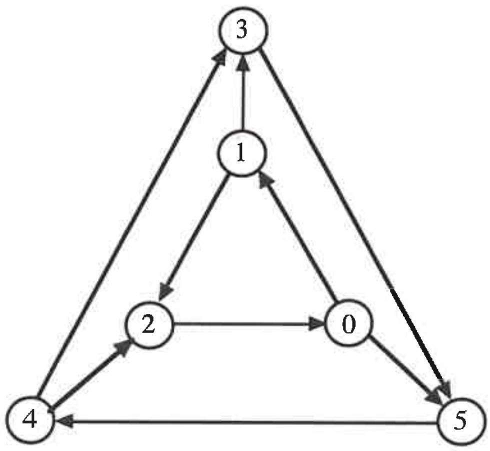
12. A topological ordering of a directed graph may be computed by:

A____ A. Ordering the vertices by descending finish time after DFS
B. Ordering the vertices by ascending discovery time after DFS
C. Ordering the vertices by ascending finish time after DFS
D. Ordering the vertices by descending discovery time after DFS

13. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?

B____ A. hashing with chaining B. open addressing
C. ordered linked list D. red-black tree

14. What is the number of strongly connected components in this graph?



A____ A. 1 B. 2 C. 3 D. 4

15. The worst-case time for Prim's algorithm implemented with a minheap is:

B____ A. $\Theta(V + E)$ B. $\Theta(E \log V)$ C. $\Theta(V \log V)$ D. $\Theta(V \log E)$

16. Suppose the compressed adjacency list representation is used for a directed graph with n vertices and m edges. The value stored at the last entry of the tailTab is:

D____ A. n B. $n + 1$ C. m D. $m + 1$

17. The number of potential probe sequences when using double hashing with a table with m entries (m is prime) is:

C____ A. $O(\log m)$ B. m C. $m(m-1)$ D. $m!$

18. For which graph representation is querying for the presence of an edge supported by binary search?

D____ A. Adjacency lists (ordered) B. Adjacency lists (unordered)
C. Adjacency matrix D. Compressed adjacency lists (ordered)

19. The maximum number of rotations while inserting a key into a red-black tree is:

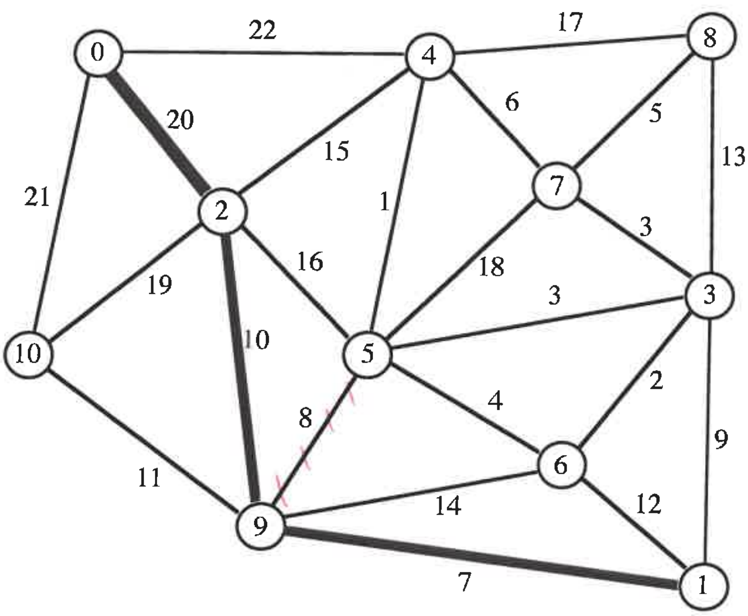
B A. 1 B. 2 C. 3 D. the black-height

20. When finding the strongly connected components, the number of components is indicated by:

D
A. The number of back edges found during the first depth-first search.
B. The number of cross edges found during the second depth-first search.
C. The number of times the vertex color table is scanned during the first depth-first search.
D. The number of times the vertex color table is scanned during the second depth-first search.

Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



Before

3	9(1)
4	15(2)
5	8(9)
6	12(1)
7	∞
8	∞
10	11(9)

After

3	3(5)
4	1(5)
6	4(5)
7	18(5)
8	∞
10	11(9)

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + cn \in \Theta(n \log$$

$$T(k) \le d\, k \log_3 k$$

$$T\left(\frac{n}{3}\right) \le d\frac{n}{3}\log_3\frac{n}{3} = d\frac{n}{3}\log_3 n - d\frac{n}{3}$$

$$T\left(\frac{2n}{3}\right) \le d\frac{2n}{3}\log_3\frac{2n}{3} = d\frac{2n}{3}\log_3 2 + d\frac{2n}{3}\log_3 n - d\frac{2n}{3}$$

$$= d\frac{2n}{3}\log_3 n + d\frac{2n}{3}(\log_3 2 - 1)$$

$$T(n) \le d\frac{n}{3}\log_3 n - d\frac{n}{3}$$

$$+ d\frac{2n}{3}\log_3 n + d\frac{2n}{3}(\log_3 2 - 1) + cn$$

$$= dn\log_3 n + d\frac{2n}{3}\log_3 2 - dn + cn$$

$$\le dn\log_3 n \quad \text{for} \quad d \ge \frac{c}{1 - \frac{2}{3}\log_3 2}$$

2. Consider the following hash table whose keys were stored by double hashing using $h_1(key) = key \% 11$ and $h_2(key) = 1 + (key \% 10)$. ***Show your work***.

```
0  22
1
2  ——— 142
3  17
4  4
5  15
6  28
7
8
9  ——— 130
10 10
```

a. Suppose 142 is to be inserted (using double hashing). Which slot will be used? (5 points)

h_1: 11)142 → 12, 11, 32, 22, (10)

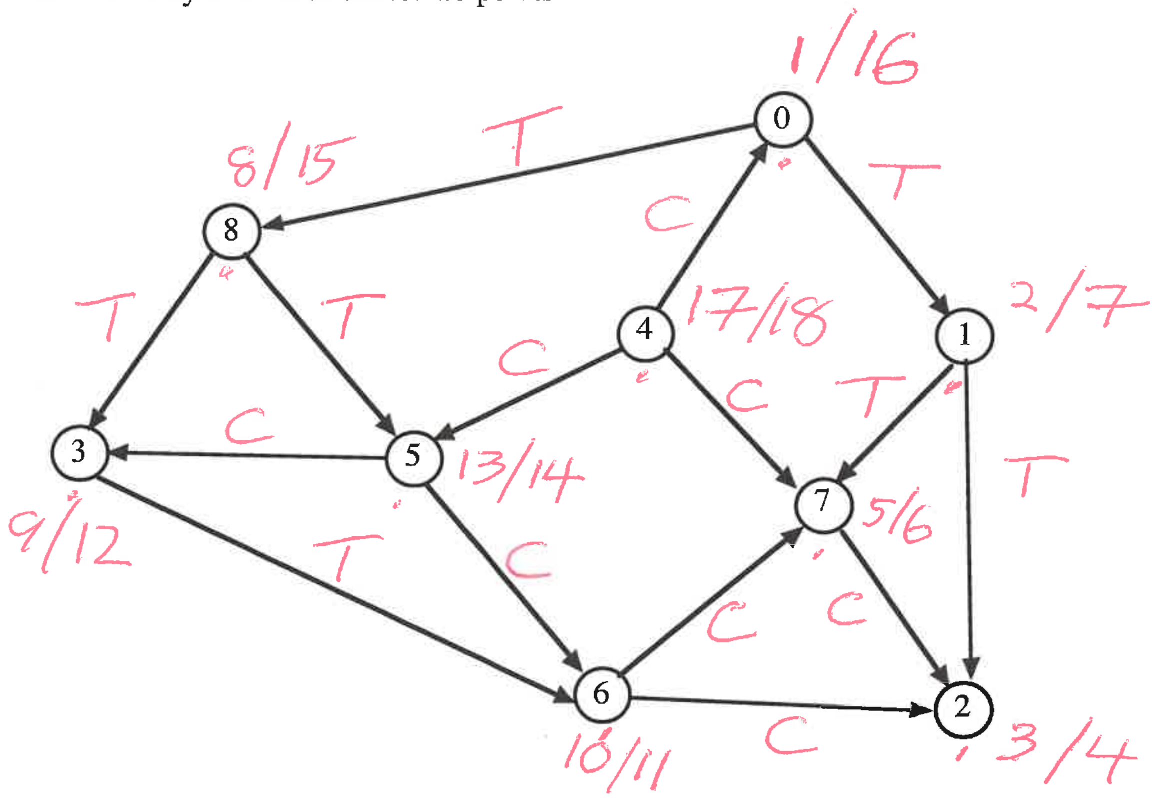
h_2: 10)142 → 14, 10, 42, 40, 2+1 → (3)

PS: 10, (2)

b. Suppose 130 is to be inserted (using double hashing) ***after*** 142 has been stored. Which slot will be used? (5 points)

h_1: 11)130 → 11, 11, 20, 11, (9)

3. Demonstrate, for the graph below, the algorithm that uses ***depth-first search*** to determine a topological ordering. Assume that the adjacency lists are ***ordered***. Show your work by labeling vertices with discovery and finish times. 10 points



Topological Ordering:

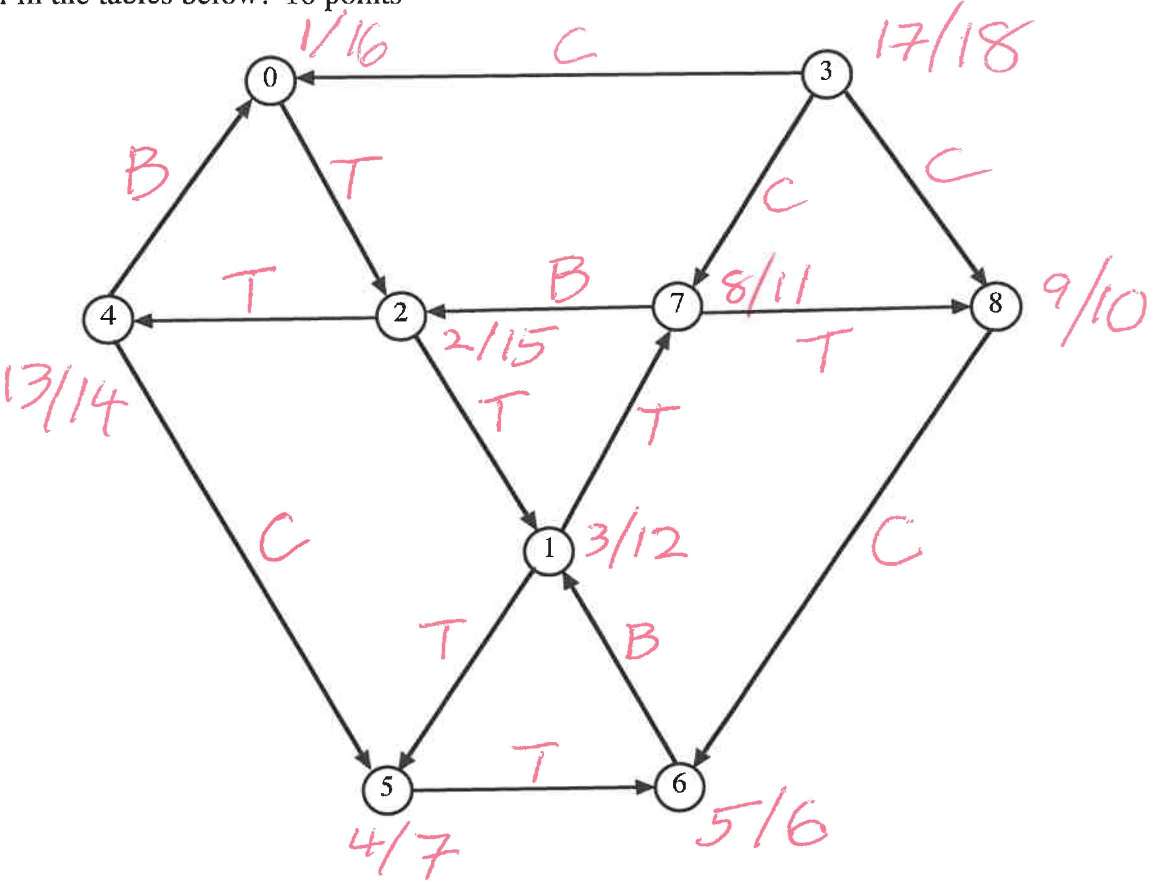
4 0 8 5 3 6 1 7 2

4. Demonstrate the Floyd-Warshall algorithm, ***with successors***, for the following input adjacency matrix. (999 represents infinity) The paths indicated in the final matrix must have ***at least one*** edge. You ***are not*** required to show the intermediate matrices. 10 points.

	0	1	2	3	4
0	999	999	11	12	4
1	999	8	6	999	5
2	20	999	15	999	999
3	999	999	5	999	999
4	999	999	3	999	4

	0	1	2	3	4
0:	27,4		7,4	12,3	4,4
1:	26,2	8,1	6,2	38,2	5,4
2:	20,0		15,2	32,0	24,0
3:	25,2		5,2	37,2	29,2
4:	23,2		3,2	35,2	4,4

5. Perform depth-first search on the following graph, including discovery/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points

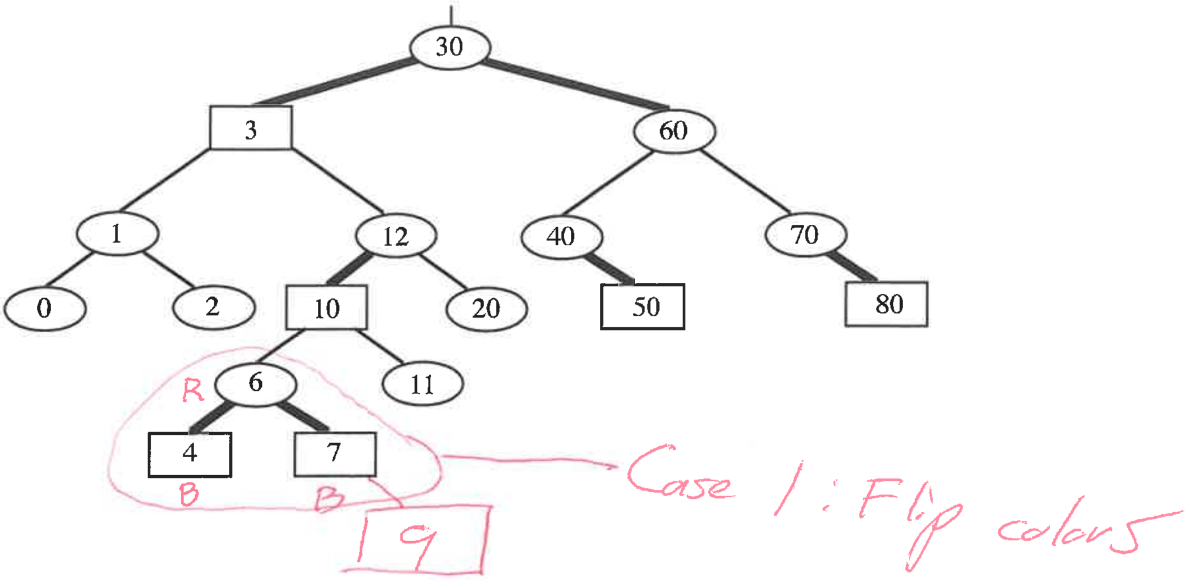


Vertex	Discovery	Finish
0	1	16
1	3	12
2	2	15
3	17	18
4	13	14
5	4	7
6	5	6
7	8	11
8	9	10

Edge	Type
0 2	T
1 5	T
1 7	T
2 1	T
2 4	T
3 0	C
3 7	C
3 8	C
4 0	B
4 5	C

Edge	Type
5 6	T
6 1	B
7 2	B
7 8	T
8 6	C

6. Insert 9 into the given red-black tree. Be sure to indicate the cases that you used (10 points)



Case 3: Rotate right at 12