CSE 2320
Test 1
Summer 2018

Key 1
Your name as it appears on your UTA ID Card

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

1. The time to run the code below is in:

```
for (i=n-1; i>=0; i-=2)
  for (j=15; j<100; j+=3)
    sum+=i+j;
```

D A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

2. A sort is said to be stable when:

B A. The expected time and the worst-case time are the same.
B. Items with the same key will appear in the same order in the output as in the input.
C. It removes duplicate copies of any key in the final output.
D. It runs in $O(n \log n)$ time.

3. Which of the following is false?

D A. $n^3 \in \Omega(n^2)$ B. $n^2 \in \Omega(n \log n)$
C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $\log\log n \in \Omega(\log n)$

4. Bottom-up maxheap construction is based on applying `maxHeapify` in the following fashion:

B A. In ascending slot number order, for each slot that is a parent.
B. In descending slot number order, for each slot that is a parent.
C. $\frac{n}{2}$ times, each time from subscript 1.
D. In descending slot number order, for each slot that is a leaf.

5. Which of the following functions is not in $\Omega(n^2)$?

C A. $n^2 \lg n$ B. n^3 C. n D. n^2

6. $f(n) = n \lg n$ is in all of the following sets, except

C A. $\Omega(\log n)$ B. $\Theta(\log(n!))$ C. $O(n)$ D. $O(n^2)$

7. The number of calls to `merge()` while performing `mergesort` on n items is in:

C A. $\Theta(\log n)$ B. $\Theta(1)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

8. The time to run the code below is in:

```
for (i=n-5; i>=5; i--)
  for (j=2; j<n; j=2*j+1)
    sum+=i+j;
```

A A. $\Theta(n \log n)$ B. $\Theta(n^2)$ C. $\Theta(n^3)$ D. $\Theta(n)$

9. Which sort takes worst-case $\Theta(n^2)$ time and is not stable?

D A. heap B. insertion C. merge D. selection

10. Suppose you are using the substitution method to establish a Θ bound on a recurrence $T(n)$ and that you already know that $T(n) \in \Omega(\log n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A A. $T(n) \in \Omega(n^3)$ B. $T(n) \in O(\log n)$ C. $T(n) \in O(n)$ D. $T(n) \in \Omega(n^2)$

11. What is n, the number of elements, for the largest table that can be processed by binary search using no more than 5 probes?

A A. 31 B. 63 C. 64 D. 127

12. Which of the following best approximates $H_m - H_n$? $(m > n)$

A A. $\ln(m/n)$ B. $\ln(m-n)$ C. H_{m-n} D. $1/(m-n)$

13. Which of the following facts can be proven using one of the limit theorems?

D A. $n^2 \in \Omega(n^3)$ B. $n^2 \in O(n \log n)$

C. $g(n) \in \Theta(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$ D. $3^n \in \Omega(2^n)$

14. $4^{\lg 7}$ evaluates to which of the following? (Recall that $\lg x = \log_2 x$.)

D A. $\sqrt{7}$ B. 7 C. 25 D. 49

15. When solving the fractional knapsack problem, the items are processed in the following order.

D A. Ascending order of weight B. Ascending order of $$$/lb
C. Descending order of weight D. Descending order of $$$/lb

16. When did we use $\sum_{k=0}^{t} x^k \le \sum_{k=0}^{\infty} x^k = \lim_{k\to\infty} \frac{x^k - 1}{x-1} = \frac{1}{1-x}$?

C A. To define H_n
B. For a recursion tree that has the same contribution for each level
C. For a recursion tree that has decreasing contributions by each level going away from the root
D. For a recursion tree that has increasing contributions by each level going away from the root

17. Which of the following is not true regarding a maxheap with 1000 elements?

A A. Subscript 1 will store the minimum priority.
B. The parent for the node with subscript 500 is stored at subscript 250.
C. The left child for the node with subscript 200 is stored at subscript 400.
D. The right child for the node with subscript 455 is stored at subscript 911.

18. The recursion tree for mergesort has which property?

A A. each level has the same contribution B. it leads to a definite geometric sum
C. it leads to a harmonic sum D. it leads to an indefinite geometric sum

19. When solving the activity scheduling problem (unweighted interval scheduling), the intervals are processed in the following order.

A A. Ascending order of finish time B. Descending order of interval length
C. Ascending order of start time D. Descending order of finish time

20. The time for the following code is in which set?

```
for (i=0; i<5; i++)
  for (j=2; j<n; j++)
  {
    c[i][j] = 0;
    for (k=0; k<n; k++)
      c[i][j] += a[i][k]*b[k][j];
  }
```

C A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

21. The goal of the Huffman coding method is:

B
A. Construct a max-heap for the symbols in an alphabet
B. Minimize the expected bits per symbol.
C. Find the symbols with high probability of occuring.
D. Maximize the compression for every string.

22. Suppose you are given a large table with n integers in descending order, possibly with repeated values. How much time is needed to determine the minimum value?

A
A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

23. The number of calls to `heapExtractMin` to build a Huffman code tree for n symbols is:

D
A. $\Theta(\log n)$ B. $n - 1$ C. n D. $2n - 2$

24. Which technique allows interfacing a priority queue with a dictionary?

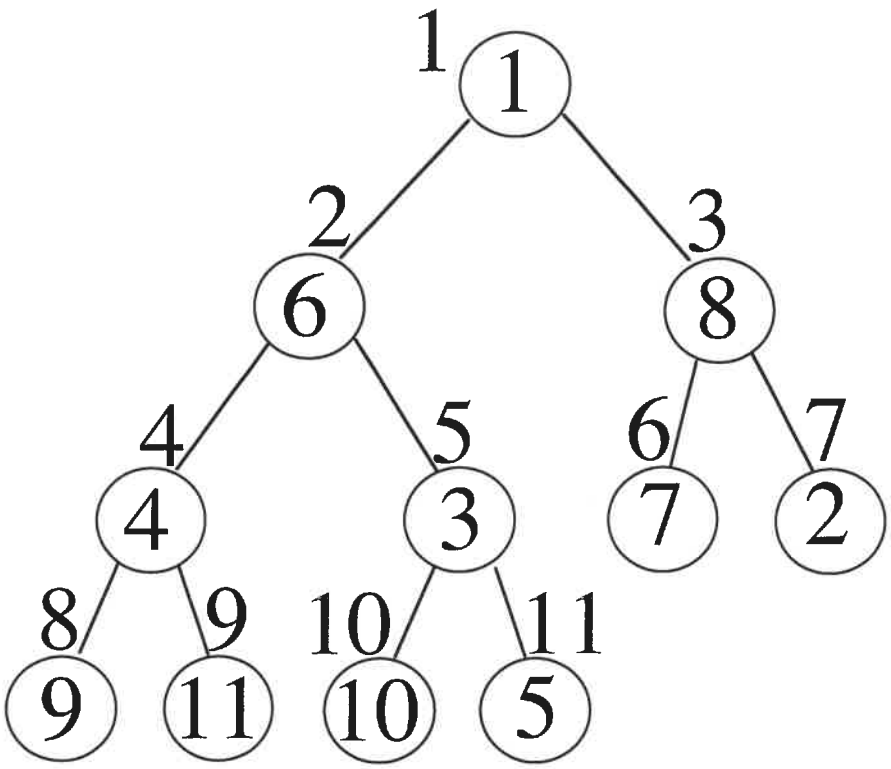
A
A. Array of handles B. Binary search tree
C. `maxHeapChange` D. Binary search

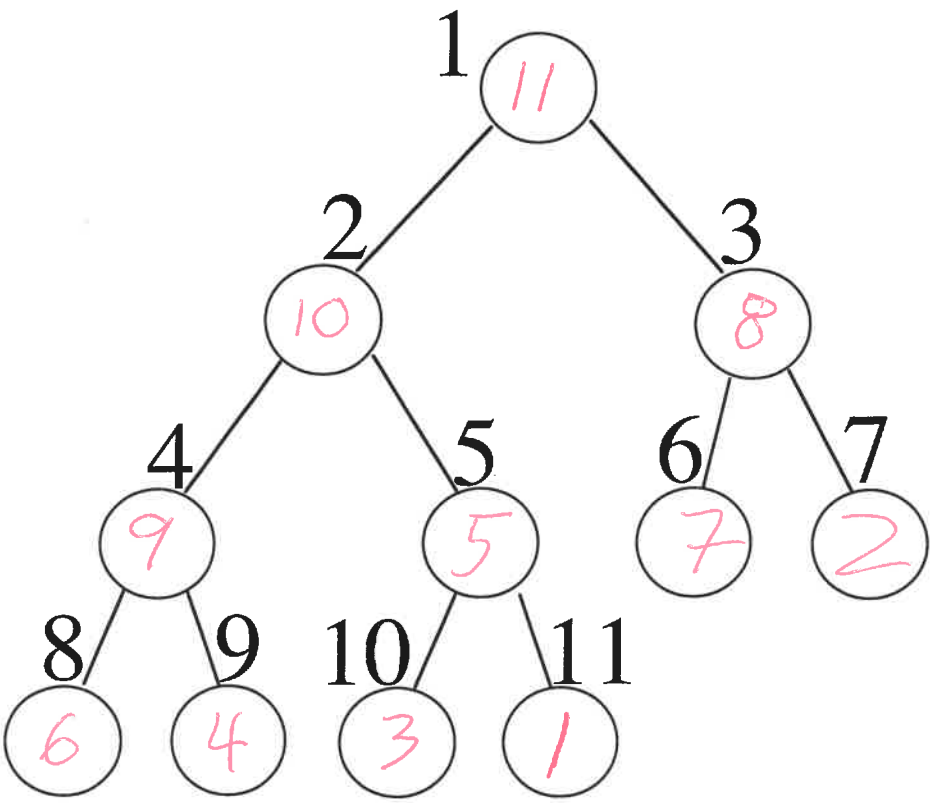
25. What is the value of $\sum_{k=0}^{t-1} 2^k$?

B
A. 2^k B. $2^t - 1$ C. $2^{t+1} - 1$ D. $2^{t+1} + 1$

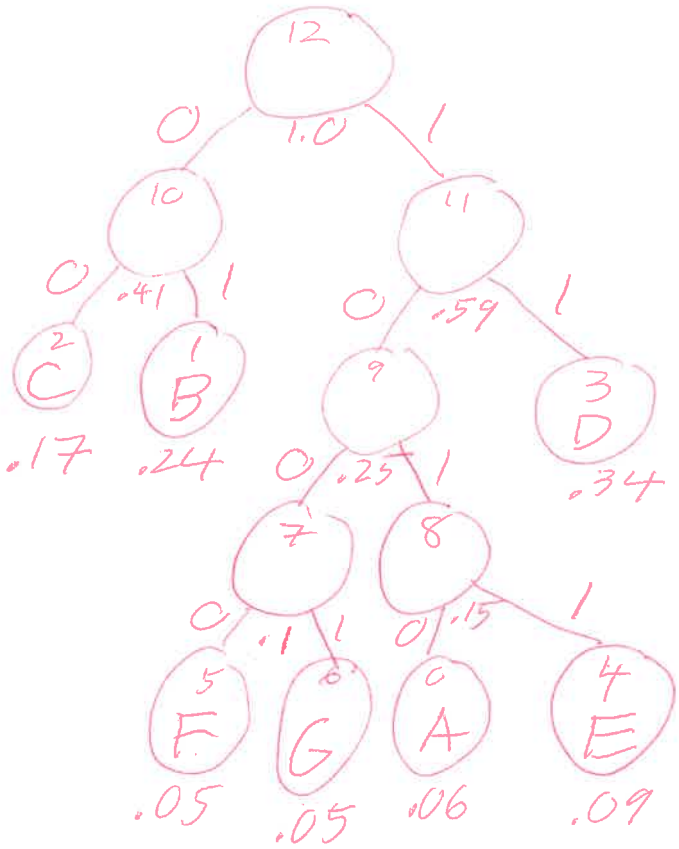
Long Answer

1. Use the efficient construction from Notes 05 to convert into a maxheap. 10 points





```
Enter alphabet size
7
.06 .24 .17 .34 .09 .05 .05
Probabilities sum to 1.000000
   i  prob  parent  bits product  code
A  0 0.060      8     4   0.240  1010
B  1 0.240     10     2   0.480  01
C  2 0.170     10     2   0.340  00
D  3 0.340     11     2   0.680  11
E  4 0.090      8     4   0.360  1011
F  5 0.050      7     4   0.200  1000
G  6 0.050      7     4   0.200  1001
Expected bits per symbol: 2.500000
   i  prob    left right parent
   7 0.100      5     6      9
   8 0.150      0     4      9
   9 0.250      7     8     11
  10 0.410      2     1     12
  11 0.590      9     3     12
  12 1.000     10    11     -1
```



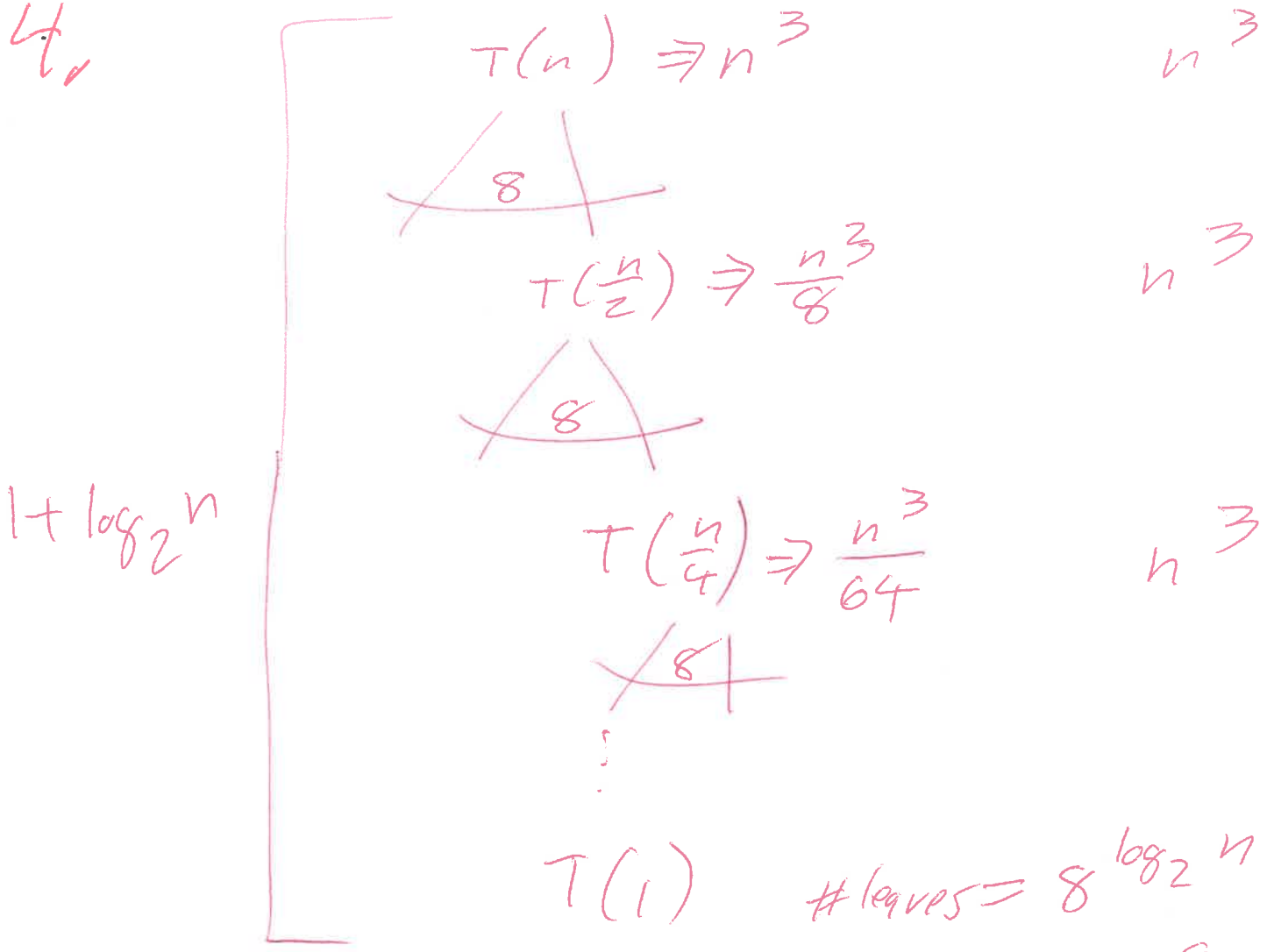
2. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

A	0.06
B	0.24
C	0.17
D	0.34
E	0.09
F	0.05
G	0.05

3. Suppose an `int` array `a` contains `m` zeroes followed by `n` ones, where `m` and `n` are unknown non-negative values. The size of the array is given to you as a non-negative value `p`, i.e. `p==m+n`. Give C code to determine `m` in $O(\log p)$ time using *binary search*. (Only the code for this task, setting the value of `m`, is needed. I/O, declarations, a `return`, etc. are unnecessary. Your code must stay within the legal subscripts for array `a`.) 10 points

```
low = 0;
high = p - 1;
while (low <= high)
{
    mid = (low + high) / 2;
    if (a[mid] == 0)
        low = mid + 1;
    else
        high = mid - 1;
}
m = low;
```

4.



$$n^3(1+\log_2 n) = \Theta(n^3 \log n)$$

4. Use the recursion-tree method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^3$ is in $\Theta\left(n^3 \log n\right)$. 10 points

4.5. Suppose $T(k) \le ck^3 \log_2 k$ for $k < n$

$$T\left(\frac{n}{2}\right) \le c\frac{n^3}{8}\log_2 \frac{n}{2}$$

$$= c\frac{n^3}{8}\log_2 n - c\frac{n^3}{8}$$

$$T(n) = 8\,T\left(\frac{n}{2}\right) + n^3$$

$$\le 8\left[c\frac{n^3}{8}\log_2 n - c\frac{n^3}{8}\right] + n^3$$

$$= cn^3\log_2 n - cn^3 + n^3$$

$$\le cn^3\log_2 n \quad \text{for} \quad c \ge 1$$

5. Use the substitution method to show that $T(n) = 8T\left(\frac{n}{2}\right) + n^3$ is in $O\left(n^3 \log n\right)$. (You do not need to show that $T(n)$ is in $\Omega\left(n^3 \log n\right)$.) 10 points