

Multiple Choice:

1. Write the letter of your answer on the line (_____) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 3 points each

1. In the example of recycling the elements of a list in $O(1)$ time, which situation holds?

- C A. Both lists are circular
B. Both lists are not circular
C. The list to be recycled is circular, the garbage list is not
D. The garbage list is circular, the list to be recycled is not

2. What is the worst-case time to perform $\text{MINIMUM}(L)$ for a sorted, doubly-linked list with n nodes?

- A A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

3. Which binary tree traversal corresponds to the following recursive code?

```
void traverse(noderef x)
{
    if (x==null)
        return;
    // process x here
    traverse(x.left);
    traverse(x.right);
}
```

- C A. inorder B. postorder C. preorder D. search for key x

4. Suppose that only numbers in $1 \dots 100$ appear as keys in a binary search tree. While searching for 50, which of the following sequences of keys could not be examined?

- D A. 10, 30, 70, 60, 50 B. 100, 20, 80, 30, 50
C. 1, 100, 20, 70, 50 D. 10, 40, 70, 30, 50

5. What does counting sort count?

- B A. the number of bytes in the input array
B. the number of occurrences for each possible key value
C. the number of different input values that have occurred
D. the maximum length among all the strings being sorted

6. Which of the following is not true regarding dynamic programming?

- A A. It is a form of divide-and-conquer
 B. It is a form of exhaustive search
 C. A cost function must be defined
 D. The backtrace may be based on recomputing the cost function

7. The time to extract the LCS (for sequences of lengths m and n) after filling in the dynamic programming matrix is in:

- B A. $\Theta(n)$ B. $\Theta(m + n)$ C. $\Theta(n \log n)$ D. $\Theta(mn)$

8. The queue for breadth-first rat-in-a-maze stores

- C A. all maze positions that have walls
 B. maze positions that must be in the final path
 C. maze positions that have been reached
 D. the current path being explored

9. For which of the following sorts does the decision tree model not apply?

- B A. Insertion B. LSD Radix Sort C. MERGE-SORT D. QUICKSORT

10. Given a pointer to a node, the worst-case time to delete the node from a singly-linked list with n nodes in ascending order is:

- D ~~A~~ A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n \log n)$ D. $\Theta(n)$

11. Memoization is associated with which technique?

- A A. top-down dynamic programming
 B. circular lists
 C. greedy methods
 D. bottom-up dynamic programming

12. If POP is implemented as $\text{return stack[--SP]}$, then PUSH of element X is implemented as:

- B A. $\text{return stack[SP++]}$ B. stack[SP++] = X C. stack[--SP] = X D. stack[++SP] = X

13. The cost function for the optimal matrix multiplication problem is:

- C A. $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k, j) + P_{i-1}P_kP_j\}$ B. $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k+1, j) + P_iP_kP_j\}$
 C. $C(i, j) = \min_{i \leq k < j} \{C(i, k) + C(k+1, j) + P_{i-1}P_kP_j\}$ D. $C(i, j) = \max\{C(i, j-1), C(i-1, j)\}$ if $x_i \neq y_j$

14. The worst-case number of comparisons for finding the k th largest of n keys using PARTITION is in which asymptotic set?

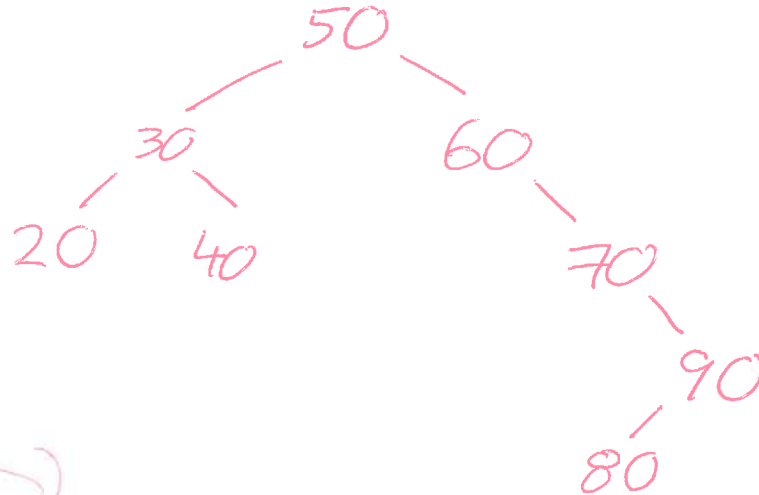
- D A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2)$

15. Suppose a (singly) linked list is used to implement a queue. Which of the following is true?

- A A. The head points to the first element and the tail points to the last element.
 B. The tail points to the first element and the head points to the last element.
 C. Like a circular queue, the maximum number of items is determined at initialization.
 D. One node is always wasted.

Long Answer

1. Give the unbalanced binary search tree that results when the keys 50, 30, 40, 60, 70, 90, 20, 80 are inserted, in the given order, into an initially empty tree. (5 points)

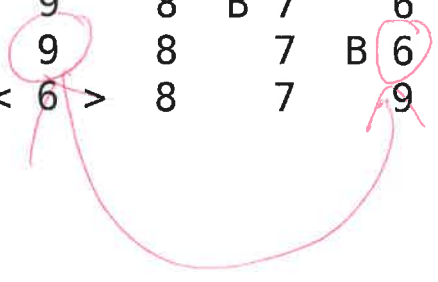


2. A billion integers in the range $0 \dots 2^{32} - 1$ will be sorted by LSD radix sort. How much faster is this done using radix $0 \dots 2^8 - 1$ rather than $0 \dots 2^4 - 1$? Show your work. (10 points)

| | |
|---|---|
| $K = 2^8$ $d = 4$ $\Theta(d(n + K))$ $\Theta(4(1B + 2^8))$ | $K = 2^4$ $d = 8$ $\Theta(8(1B + 2^4))$ |
|---|---|

↑
Twice as fast

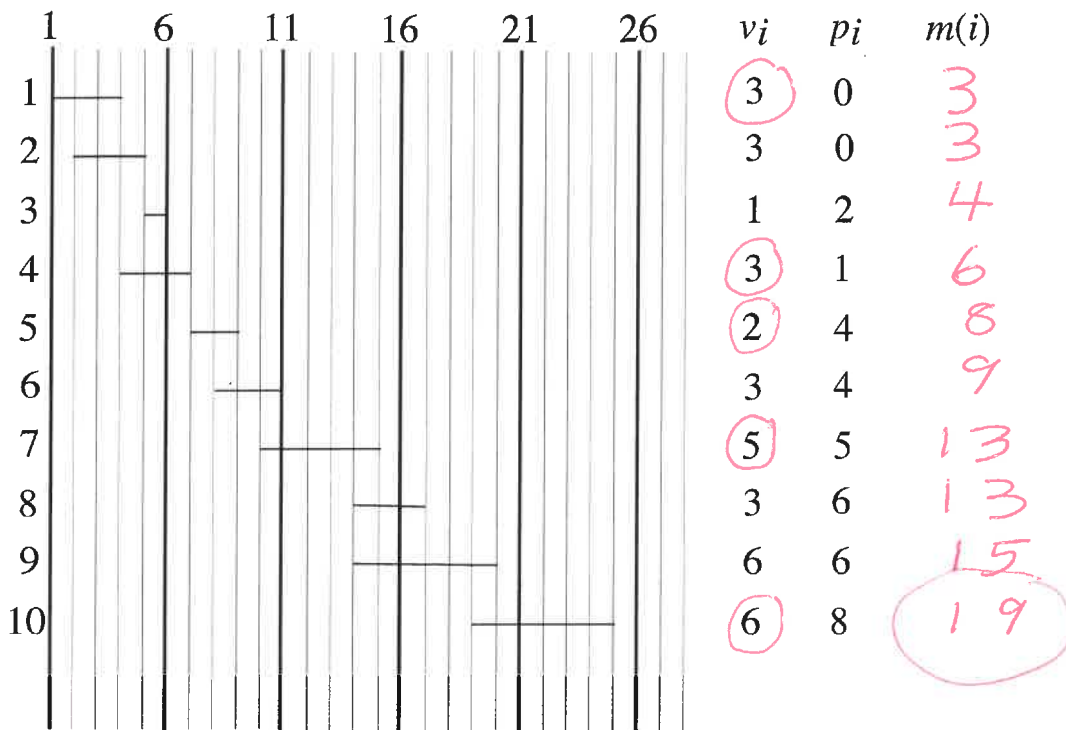
| | | | | | | | | | | | | | | | | | | | |
|----|---|---|---|---|---|---|---|---|---|---|---|---|-------|---|---|---|---|---|---|
| AB | 8 | | 2 | | 5 | | 3 | | 4 | | 1 | | 9 | | 0 | | 7 | | 6 |
| A | 8 | B | 2 | | 5 | | 3 | | 4 | | 1 | | 9 | | 0 | | 7 | | 6 |
| | 2 | A | 8 | B | 5 | | 3 | | 4 | | 1 | | 9 | | 0 | | 7 | | 6 |
| | 2 | | 5 | A | 8 | B | 3 | | 4 | | 1 | | 9 | | 0 | | 7 | | 6 |
| | 2 | | 5 | | 3 | A | 8 | B | 4 | | 1 | | 9 | | 0 | | 7 | | 6 |
| | 2 | | 5 | | 3 | | 4 | A | 8 | B | 1 | | 9 | | 0 | | 7 | | 6 |
| | 2 | | 5 | | 3 | | 4 | | 1 | A | 8 | B | 9 | | 0 | | 7 | | 6 |
| | 2 | | 5 | | 3 | | 4 | | 1 | A | 8 | | 9 | B | 0 | | 7 | | 6 |
| | 2 | | 5 | | 3 | | 4 | | 1 | | 0 | A | 9 | | 8 | B | 7 | | 6 |
| | 2 | | 5 | | 3 | | 4 | | 1 | | 0 | A | 9 | | 8 | | 7 | B | 6 |
| | 2 | | 5 | | 3 | | 4 | | 1 | | 0 | | < 6 > | | 8 | | 7 | | 9 |



3. Show the result after PARTITION (Version 1) manipulates the following subarray. Recall that both pointers start at the left end of the subarray. (10 points)

8 2 5 3 4 1 9 0 7 6

4. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. (10 points)



| i | S |
|---|----|
| 0 | 0 |
| 1 | 2 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |
| 5 | 11 |
| i | C |

| | |
|----|---|
| 0 | 0 |
| 1 | 6 |
| 2 | 1 |
| 3 | 2 |
| 4 | 6 |
| 5 | 2 |
| 6 | 6 |
| 7 | 3 |
| 8 | 3 |
| 9 | 4 |
| 10 | 3 |
| 11 | 5 |
| 12 | 4 |
| 13 | 5 |
| 14 | 4 |
| 15 | 4 |
| 16 | 5 |
| 17 | 4 |
| 18 | 5 |

Solution

| i | S |
|---|----|
| 5 | 11 |
| 3 | 5 |
| 1 | 2 |

5. Use the dynamic programming solution for subset sums to determine a subset that sums to 18. Be sure to give the complete table that would be produced. (10 points)

| | | | | | | |
|-------|---|---|---|---|---|----|
| i | 0 | 1 | 2 | 3 | 4 | 5 |
| S_i | 0 | 2 | 3 | 5 | 7 | 11 |

6. Complete the following example of the efficient dynamic programming technique for finding a longest common subsequence. Be sure to provide the backtrace for your LCS using arrows in the matrix. (10 points)

1 0 0 1 0 1
0 1 1 0 1 0

0 1 1 0 1 0

0 0 0 0 0 0 0

1 0 0 1 1 1 1 1

0 0 1 1 1 2 2 2

0 0 1 1 1 2 2 3

1 0 1 2 2 2 3 3

0 0 1 2 2 3 3 4

1 0 1 2 3 3 4 4

100101

011010

LCS is 1010, length==4

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 1 | 0 | 1 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 0 | 0 | 1 | 1 | 1 | 2 | 2 | 3 |
| 1 | 0 | 1 | 2 | 2 | 2 | 3 | 3 |
| 0 | 0 | 1 | 2 | 2 | 3 | 3 | 4 |
| 1 | 0 | 1 | 2 | 3 | 3 | 4 | 4 |