

Multiple Choice:

1. Write the letter or value for your answer on the line ( \_\_\_\_\_ ) to the LEFT of each problem.
2. CIRCLED ANSWERS DO NOT COUNT.
3. 2 points each

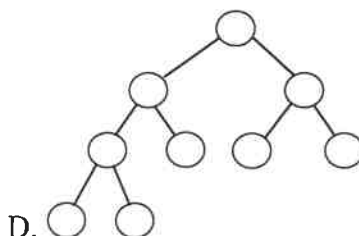
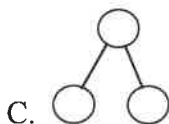
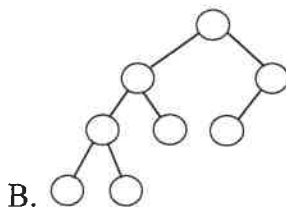
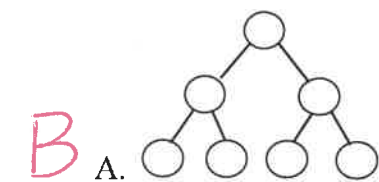
1. Suppose the compressed adjacency list representation is used for a directed graph with  $n$  vertices and  $m$  edges. The last subscript for the tailTab is:

A A.  $n$       B.  $n + 1$       C.  $m$       D.  $m + 1$

2. The expected number of probes for a an unsuccessful search in hashing by chaining with  $\alpha$  as the load factor is:

A A.  $\alpha$       B.  $2\alpha$       C.  $\frac{\alpha}{2}$       D.  $\frac{2}{3}\alpha$

3. Which of the following binary trees has *exactly* one legal coloring as a red-black tree?



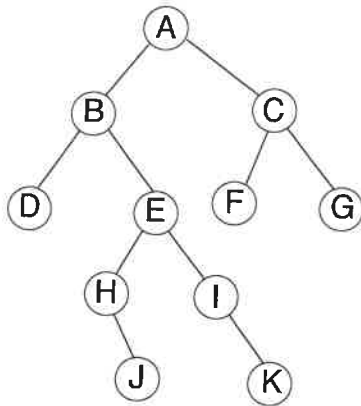
4. Which of the following cannot occur when additional edges are included in a directed graph?

C A. The number of strong components may remain the same.  
B. The number of strong components may decrease.  
C. The number of strong components may increase.  
D. The graph acquires a cycle.

5. For a double hash table with  $\alpha = 0.8$  (without deletions), the upper bound on the expected number of probes for unsuccessful search is:

5

6. Suppose the tree below is a binary search tree whose keys and subtree sizes are not shown. Which node will contain the key with rank 8? (Write the node's letter on the line.)



A

7. What is required when calling `union(i, j)` for maintaining disjoint subsets?

C

- A.  $i$  and  $j$  are leaders for the same subset  
 B.  $i$  and  $j$  are in the same subset  
 C.  $i$  and  $j$  are leaders for different subsets  
 D.  $i$  is the ancestor of  $j$  in one of the trees

8. During a breadth-first search, the status of a gray vertex is:

B

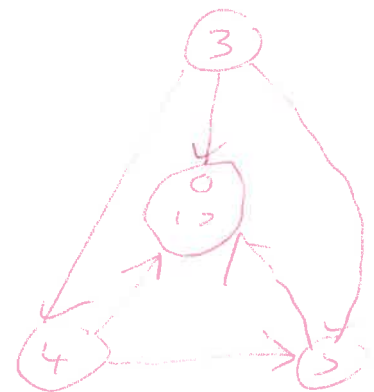
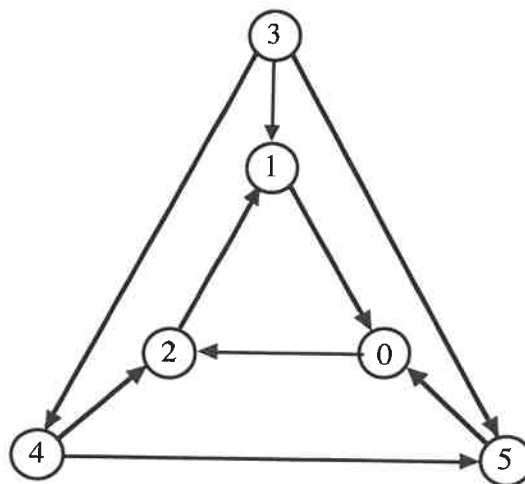
- A. It has been completely processed.  
 B. It is in the FIFO queue.  
 C. It is in the priority queue.  
 D. It is undiscovered.

9. The cycle property for minimum spanning trees may be used to find an MST by:

C

- A. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.  
 B. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.  
 C. Remove the maximum weight edge in any cycle until only a tree of edges remains.  
 D. Remove the minimum weight edge in any cycle until only a tree of edges remains.

10. What is the number of strongly connected components in this graph?



4

11. Which algorithm maintains multiple subtrees?

B

A. Dijkstra's

B. Kruskal's

C. Prim's

D. Warshall's

12. A topological ordering of a directed graph may be computed by:

A

A. Ordering the vertices by descending finish time after DFS

B. Ordering the vertices by ascending discovery time after DFS

C. Ordering the vertices by ascending finish time after DFS

D. Ordering the vertices by descending discovery time after DFS

13. Using the values *never-used* (-1) and *recycled* (-2) are part of which data structure?

B

A. hashing with chaining

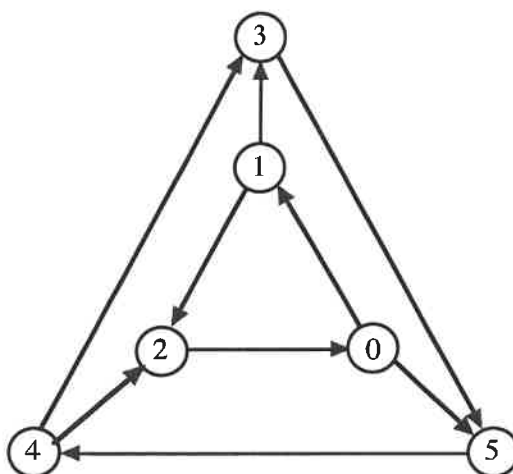
B. open addressing

C. ordered linked list

D. red-black tree

14. What is the number of strongly connected components in this graph?

1



15. The worst-case time for Prim's algorithm implemented with a minheap is:

B

A.  $\theta(V + E)$

B.  $\theta(E \log V)$

C.  $\theta(V \log V)$

D.  $\theta(V \log E)$

16. Suppose the compressed adjacency list representation is used for a directed graph with  $n$  vertices and  $m$  edges. The value stored at the last entry of the tailTab is:

C

A.  $n$

B.  $n + 1$

C.  $m$

D.  $m + 1$

17. The number of potential probe sequences when using double hashing with a table with  $m$  entries ( $m$  is prime) is:

C

A.  $O(\log m)$

B.  $m$

C.  $m(m - 1)$

D.  $m!$

18. For which graph representation is querying for the presence of an edge supported by binary search?

- D A. Adjacency lists (ordered) B. Adjacency lists (unordered)  
C. Adjacency matrix D. Compressed adjacency lists (ordered)

19. The maximum number of rotations while inserting a key into a red-black tree is:

- B A. 1 B. 2 C. 3 D. the black-height

20. Suppose a node  $x$  in an unbalanced binary search tree has two children, each storing one key. What is the first step to delete  $x$ ?

- A A. Find the successor of  $x$  B. Inorder traversal  
C. Rotate  $x$  so it becomes a leaf D. Splice the parent of  $x$  to either child of  $x$

21. Suppose a depth-first search on a directed graph yields a path of tree edges from vertex  $X$  to vertex  $Y$  and a path of tree edges from vertex  $X$  to  $Z$ . If there is also an edge from  $Y$  to  $Z$ , then its type will be:

- C A. Tree B. Back C. Cross D. Forward



22. During depth-first search on an undirected graph, a cycle is indicated by which edge type?

- A A. Back B. Cross C. Forward D. Tree

23. Which edge is chosen in a phase of Kruskal's algorithm?

- D A. The unprocessed edge  $(x, y)$  of smallest weight such that  $\text{find}(x) == \text{find}(y)$   
B. An edge of maximum-weight in a cycle (to be excluded)  
C. An edge that is on a shortest path from the source  
D. The unprocessed edge  $(x, y)$  of smallest weight such that  $\text{find}(x) \neq \text{find}(y)$

Problems 24 and 25 refer to the following hash table whose keys are stored by linear probing using  $h(\text{key}) = \text{key} \% 13$ .

| 0 | 1 | 2 | 3  | 4 | 5   | 6   | 7  | 8  | 9  | 10 | 11 | 12 |
|---|---|---|----|---|-----|-----|----|----|----|----|----|----|
|   |   |   | 94 |   | 122 | 110 | 20 | 86 | 87 | 62 |    |    |

24. 148 would be inserted into which slot of the given table?

- 11 A. 0 B. 1 C. 2 D. 4 E. 11 F. 12

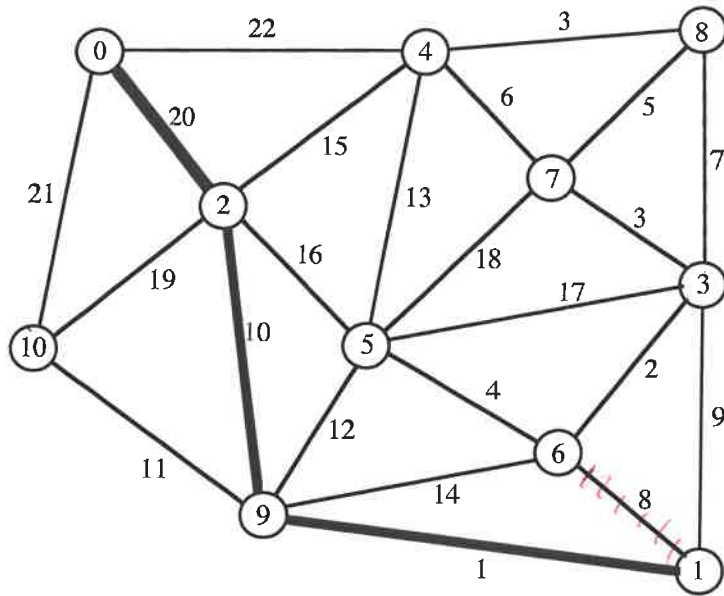
$$\begin{array}{r} 11 \\ 13 \overline{) 148} \\ \underline{13} \phantom{0} \\ 18 \\ \underline{13} \\ 5 \end{array}$$

25. 133 would be inserted into which slot of the given table?

- 4 A. 0 B. 1 C. 2 D. 4 E. 11 F. 12

## Long Answer

1. What are the entries in the heap (for Prim's algorithm) before and after moving the next vertex and edge into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. Edges currently not in the MST are the narrow ones. You do not need to show the binary tree for the heap ordering. 10 points.



Before

|              |    |          |
|--------------|----|----------|
| <del>4</del> | 3  | 9(1)     |
| <del>5</del> | 4  | 15(2)    |
|              | 5  | 12(9)    |
|              | 6  | 8(1)     |
|              | 7  | $\infty$ |
|              | 8  | $\infty$ |
|              | 10 | 11(9)    |

After

|    |          |
|----|----------|
| 3  | 2(6)     |
| 4  | 15(2)    |
| 5  | 4(6)     |
| 7  | $\infty$ |
| 8  | $\infty$ |
| 10 | 11(9)    |

2. Consider the following hash table whose keys were stored by double hashing using  $h_1(\text{key}) = \text{key} \% 17$  and  $h_2(\text{key}) = 1 + (\text{key} \% 16)$ .

|    |     |
|----|-----|
| 0  | -1  |
| 1  | 800 |
| 2  | -1  |
| 3  | -1  |
| 4  | 701 |
| 5  | -1  |
| 6  | 601 |
| 7  | -1  |
| 8  | 501 |
| 9  | -1  |
| 10 | 401 |
| 11 | -1  |
| 12 | 301 |
| 13 | -1  |
| 14 | 201 |
| 15 | -1  |
| 16 | 101 |

← 2001

← 2002

- a. Suppose 2001 is to be inserted (using double hashing). Which slot will be used? (5 points)

$$\begin{array}{r} 117 \\ 17 \overline{) 2001} \\ \underline{17} \\ 30 \\ \underline{17} \\ 131 \\ \underline{119} \\ 12 = h_1 \end{array}$$

$$\begin{array}{r} 125 \\ 16 \overline{) 2001} \\ \underline{16} \\ 40 \\ \underline{32} \\ 81 \\ \underline{80} \\ 1 = h_2 \end{array}$$

$1+1=2=h_2$   
12, 14, 16, 1, (3)

- b. Suppose 2002 is to be inserted (using double hashing) after 2001 has been stored. Which slot will be used? (5 points)

$$\begin{array}{r} 117 \\ 17 \overline{) 2002} \\ \underline{17} \\ 30 \\ \underline{17} \\ 132 \\ \underline{119} \\ 13 = h_1 \end{array}$$

(13)

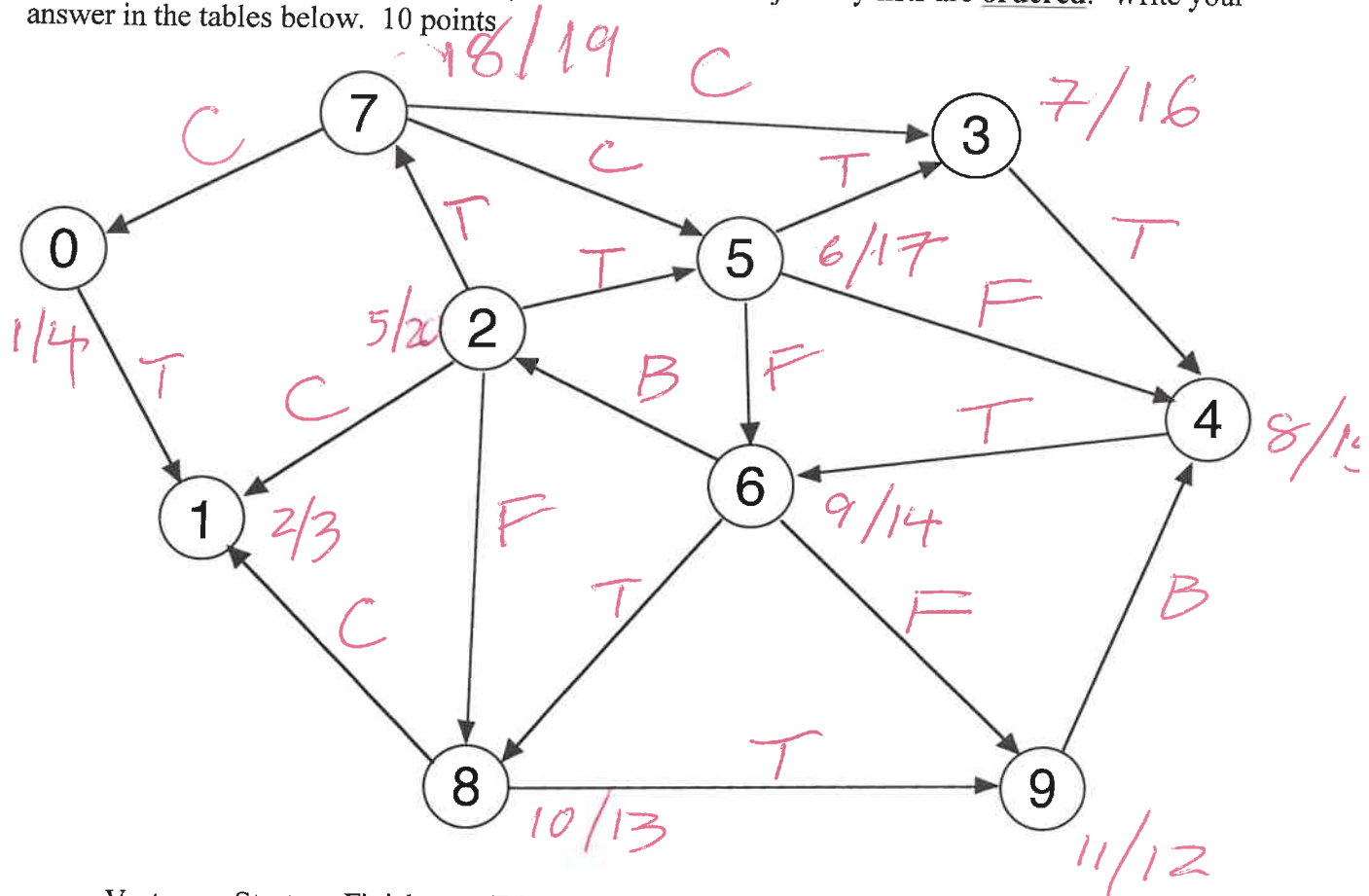
3. Demonstrate the Floyd-Warshall algorithm, with successors, for the following input adjacency matrix. (999 represents infinity) The paths indicated in the final matrix must have at least one edge. You are not required to show the intermediate matrices. 10 points.

|   | 0   | 1   | 2  | 3   | 4   |
|---|-----|-----|----|-----|-----|
| 0 | 999 | 999 | 11 | 12  | 4   |
| 1 | 999 | 8   | 6  | 999 | 5   |
| 2 | 20  | 999 | 15 | 999 | 999 |
| 3 | 999 | 999 | 5  | 999 | 999 |
| 4 | 999 | 999 | 3  | 999 | 4   |

|   | 0    | 1        | 2    | 3    | 4    |
|---|------|----------|------|------|------|
| 0 | 27 4 | $\infty$ | 7 4  | 12 3 | 4 4  |
| 1 | 26 2 | 8 1      | 6 2  | 38 2 | 5 4  |
| 2 | 20 0 | $\infty$ | 15 2 | 32 0 | 24 0 |
| 3 | 25 2 | $\infty$ | 5 2  | 37 2 | 29 2 |
| 4 | 23 2 | $\infty$ | 3 2  | 35 2 | 4 4  |

3 wrong = 1 point

4. Perform depth-first search on the following graph, including discovery/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are **ordered**. Write your answer in the tables below. 10 points



| Vertex | Start     | Finish    | Edge | Type     | Edge | Type     |
|--------|-----------|-----------|------|----------|------|----------|
| 0      | <u>1</u>  | <u>4</u>  | 0 1  | <u>T</u> | 6 2  | <u>B</u> |
| 1      | <u>2</u>  | <u>3</u>  | 2 1  | <u>C</u> | 6 8  | <u>T</u> |
| 2      | <u>5</u>  | <u>20</u> | 2 5  | <u>T</u> | 6 9  | <u>F</u> |
| 3      | <u>7</u>  | <u>16</u> | 2 7  | <u>T</u> | 7 0  | <u>C</u> |
| 4      | <u>8</u>  | <u>15</u> | 2 8  | <u>F</u> | 7 3  | <u>C</u> |
| 5      | <u>6</u>  | <u>17</u> | 3 4  | <u>T</u> | 7 5  | <u>C</u> |
| 6      | <u>9</u>  | <u>14</u> | 4 6  | <u>T</u> | 8 1  | <u>C</u> |
| 7      | <u>18</u> | <u>19</u> | 5 3  | <u>T</u> | 8 9  | <u>T</u> |
| 8      | <u>10</u> | <u>13</u> | 5 4  | <u>F</u> | 9 4  | <u>B</u> |
| 9      | <u>11</u> | <u>12</u> | 5 6  | <u>F</u> |      |          |

3 incorrect = 1 point



5. Insert 9 into the given red-black tree. Be sure to indicate the cases that you used (10 points)

