CSE 2320

Name

Test 1

Fall 2010

Last 4 Digits of Mav ID # ____ Multiple Choice. Write your answer to the LEFT of each problem. 3 points each

1. The time to convert an array, with priorities stored at subscripts 1 through n, to a minheap is in:

A.
$$\Theta(n)$$
 B. $\Theta(\log n)$ C. $\Theta(n^3)$ D. $\Theta(n \log n)$

2. The number of calls to mergeAB while performing mergesort on *n* items is: C. $\Theta(n)$ A. $\Theta(\log n)$ B. $\Theta(m+n)$ D. $\Theta(n \log n)$

3. Which of the following is not true? A. $n^2 \in O(n^3)$ B. $n \log n \in \Omega(n^2)$ C. $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Omega(g(n))$ D. $3^n \in \Omega(2^n)$

4. The cost function for the optimal matrix multiplication problem is:

$$\begin{array}{ll} \text{A. } C(i,j) = \min_{i \le k < j} \left\{ C(i,k) + C(k,j) + P_{i-1}P_kP_j \right\} & \text{B. } C(i,j) = \min_{i \le k < j} \left\{ C(i,k) + C(k+1,j) + P_iP_kP_j \right\} \\ \text{C. } C(i,j) = \min_{i \le k < j} \left\{ C(i,k) + C(k+1,j) + P_{i-1}P_kP_j \right\} & \text{D. } C(i,j) = \max \left\{ C(i,j-1), C(i-1,j) \right\} \text{ if } x_i \neq y_j \\ \end{array}$$

5. The function $n + 3n^2 \log n$ is in which set?

A.
$$\Omega(n^2)$$
 B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$

6. $f(n) = n \lg n$ is in all of the following sets, except

A.
$$O(\log n)$$
 B. $\Theta(\log(n!))$ C. $\Omega(\frac{1}{n})$ D. $O(n^2)$

7. Which statement is correct regarding the unweighted and weighted activity scheduling problems?

- A. Both are easily solved using a greedy technique
- B. Unweighted is solved using a greedy technique, weighted is solved by dynamic programming
- C. Both require dynamic programming
- D. Weighted is solved using a greedy technique, unweighted is solved by dynamic programming
- 8. What is the value of $\sum_{k=0}^{l} 2^k$?

A.
$$2^k$$
 B. 2^t C. $2^{t+1} - 1$ D. $2^{t+1} + 1$

9. Suppose you have correctly determined some c and n_0 to prove $f(n) \in O(g(n))$. Which of the following is not necessarily true?

A. c may be increased B. n_0 may be decreased C. n_0 may be increased D. $g(n) \in \Omega(f(n))$ 10. Suppose you are using the substitution method to establish a Θ bound on a recurrence T(n) and that

you already know that $T(n) \in \Omega(\lg n)$ and $T(n) \in O(n^2)$. Which of the following cannot be shown as an improvement?

A.
$$T(n) \in O(\lg n)$$
 B. $T(n) \in O(n)$ C. $T(n) \in \Omega(n^2)$ D. $T(n) \in \Omega(n^3)$

- 11. The time to find the maximum of the *n* elements of an integer array is in:
 - A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

12. Which sort takes worst-case $\Theta(n^2)$ time and is not stable?

C. merge D. selection A. heap B. insertion

Short answer. 3 points each

- 1. Give the exact value of H_4 .
- 2. What is *n*, the number of elements, for the largest table that can be processed by binary search using no more than 10 probes?
- Give the subscripts for the parent, left child, and right child for the maxheap element stored at subscript 455. The heap is currently storing 1000 elements in a table with 2000 slots.
 Long Answer

m(i)

- 1. Give the definition of Ω . 5 points
- 2. Use dynamic programming to solve the following instance of weighted interval scheduling. Be sure to indicate the intervals in your solution and the sum achieved. 10 points



- 3. Use the recursion-tree method to show that $T(n) = 2T\left(\frac{n}{4}\right) + 2$ is in $\Theta(\sqrt{n})$. 10 points
- 4. Use the substitution method to show that $T(n) = 2T\left(\frac{n}{4}\right) + 2$ is in $\Theta(\sqrt{n})$. 10 points
- 5. Give a Huffman code tree for the following symbols and probabilities. Besides the tree, be sure to compute the expected bits per symbol. 10 points

А		0.15	
В		0.03	
С		0.5	
D		0.03	
Е		0.22	
F		0.07	
	~1		

6. a. Show the maxheap after performing getmax. 5 points



b. Show the minheap after changing the priority at subscript 9 to 1. 5 points



- 1. If POP is implemented as return stack[--SP], then PUSH of element X is implemented as: A. return stack[SP++] B. stack[SP++] = X C. stack[SP--] = X D. stack[++SP] = X
- 2. Suppose the tree below is a binary search tree whose keys <u>are not shown</u>. Which node will contain the key that is the predecessor of the key stored at K?



- A. **A**
- B. E
- C. F
- D. I
- 3. Which of the following is a longest common subsequence for 1 2 0 1 2 0 and 0 0 1 1 2 2?
 A. 0 0 1 1
 B. 1 1 2
 C. 0 0 1 2
 D. 0 1 2 0
- 4. What is the worst-case time to perform MINIMUM(L) for an unsorted, doubly-linked list with *n* nodes? A. $\Theta(1)$ B. $\Theta(\log n)$ C. $\Theta(n)$ D. $\Theta(n \log n)$
- 5. In the example of recycling the elements of a list in O(1) time, which situation holds?
 - A. Both lists are circular
 - B. Both lists are not circular
 - C. The list to be recycled is circular, the garbage list is not
 - D. The garbage list is circular, the list to be recycled is not
- 6. The purpose of the binary searches used when solving the longest (monotone) increasing subsequence (LIS) problem is:
 - A. to search a table that will contain only the LIS elements at termination
 - B. to assure that the final solution is free of duplicate values
 - C. to determine the longest possible increasing subsequence terminated by a particular input value
 - D. to sort the original input
- 7. An array with 150 unique elements is subscripted starting with 0. You would like to iteratively use PARTITION to find the thirty largest values, but there is no requirement that the thirty largest values be ordered. Which of the following is not correct?

- A. If 120 is returned from PARTITION, we must continue.
- B. If 131 is returned from PARTITION, we must continue.
- C. If 118 is returned from PARTITION, we must continue.
- D. If 119 is returned from PARTITION, we are done.
- 8. Recursion is often an alternative to using which data structure?
- A. Linked list B. Queue C. Stack D. 2-d array
- 9. What is the worst-case time to perform a rotation in an unbalanced binary search tree storing n keys given a pointer to a node and the direction of the rotation? Assume that parent pointers are available.
 A. Θ(1) B. Θ(log n) C. Θ(n) D. Θ(n log n)
- 10. The stack for rat-in-a-maze stores
 - A. the shortest known path that leads to the cheese
 - B. all positions that have walls
 - C. maze positions that must be in the final path
 - D. the current path being explored
- 11. The two mandatory pointers in a node for a rooted tree with linked siblings are:
 - A. First child and right sibling
 - B. Left child and right child
 - C. Left child and parent
 - D. Left sibling and right sibling
- 12. Which of the following will not be true regarding the decision tree for MERGESORT for sorting *n* input values?
 - A. There will be a path from the root to a leaf with $\Omega(n^2)$ decisions.
 - B. There will be *n*! leaves.
 - C. Every path from the root to a leaf will have $O(n^2)$ decisions.
 - D. The height of the tree is $\Omega(n \log n)$.
- 13. In the example of concatenating two strings in O(1) time, which situation holds?
 - A. Both lists are circular
 - B. Both lists are not circular
 - C. The first list is not circular, the list to be attached is circular
 - D. The first list is circular, the list to be attached is not
- 14. Which binary tree traversal corresponds to the following recursive code? void traverse(noderef x)

```
{
if (x==null)
  return;
traverse(x.left);
traverse(x.right);
// process x here
}
```

A. inorder B. postorder C. preorder D. search for key x

- 15. Suppose that only numbers in 1 . . . 1000 appear as keys in a binary search tree. While searching for 500, which of the following sequences of keys could not be examined?
 - A. 10, 200, 300, 100, 500
 - B. 100, 1000, 200, 900, 300, 800, 400, 700, 500
 - C. 200, 300, 400, 700, 600, 500
 - D. 600, 100, 550, 540, 500

Long Answer

1. Show the binary search tree that results from performing a left rotation at the node with key 5. (5 points)



2. Give the inorder, postorder, and preorder traversals of the given binary tree. Be sure to label your traversals appropriately. (10 points)



3. Show the result after PARTITION manipulates the following subarray. Be sure to indicate which version of PARTITION you applied. (10 points)

1 8 2 5 3 7 4 9 0 6 Version:

- 4. A billion integers in the range 0 . . . 999,999 are to be sorted by LSD radix sort. How much faster will this be done if radix 0 . . . 999 is used rather than radix 0 . . . 99? Show your work. (10 points)
- 5. Use the dynamic programming solution for subset sums to determine a subset that sums to 15. (10 points)

i	0	1	2	3	4	5
S_i	0	2	3	5	7	11

6. The following binary search tree includes keys and subtree sizes. Clearly circle those nodes that would be examined while (efficiently) determining the rank of key 140. (10 points)



CSE 2320 Test 3

Fall 2010

Name _____

Last 4 Digits of Student ID # ____

Multiple Choice. Write the letter of your answer to the LEFT of each problem. 2 points each

1. The worst-case time for Prim's algorithm implemented with a T-table is:

2. Which of the following binary trees can be legally colored as a red-black tree with its root colored red?



7. The worst-case time for the memoryless version of Dijkstra's algorithm is:

A. $\theta(V^2 + E)$ B. $\theta(E \log V)$ C. $\theta(V \log V)$ D. $\theta(EV)$

- 8. An adjacency matrix is the most useful representation for which problem?
 - A. Breadth-first search
 - B. Warshall's algorithm
 - C. Depth-first search
 - D. Finding strongly-connected components
- 9. During a breadth-first search, the status of a gray vertex is:
 - A. It has been completely processed.
 - B. It is in the FIFO queue.
 - C. It is in the priority queue.
 - D. It is undiscovered.
- 10. When a graph is dense, the best way to find a minimum spanning tree is:
 - A. Floyd-Warshall algorithm
 - B. Prim's algorithm using heap
 - C. Prim's algorithm using T-table
 - D. Warshall's algorithm
- 11. A topological ordering of a directed graph may be computed by:
 - A. Ordering the vertices by ascending finish time after DFS
 - B. Ordering the vertices by descending discovery time after DFS
 - C. Ordering the vertices by ascending discovery time after DFS
 - D. Ordering the vertices by descending finish time after DFS
- 12. When finding the strongly connected components, the number of components is indicated by:
 - A. The number of back edges found during the first depth-first search.
 - B. The number of cross edges found during the second depth-first search.

- C. The number of times the vertex color table is scanned during the first depth-first search.
- D. The number of times the vertex color table is scanned during the second depth-first search.
- 13. What is the purpose of the first depth-first search when finding strongly connected components?
 - A. To assure that two vertices that are in the same cycle will be output in the same component
 - B. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in the same component.
 - C. To assure that two vertices, X and Y, with paths from X to Y but not from Y to X, are output in different components.
 - D. To make sure that the input graph has no cycles.
- 14. The number of potential probe sequences when using linear probing with a table with *m* entries is:

A.
$$m$$
 B. $m + 1$ C. $m(m-1)$ D. $m!$

15. What is the number of strongly connected components in this graph?



16. The following matrix was produced by Warshall's algorithm with successors. How many vertices are on the represented path from 0 to 1?

D. 4

- -1 3 3 3 3 -1 3 3 3 4 -1 1 1 1 4 2 2 2 2 -1 -1 -1 -1 -1 -1 A. 1 B. 2 C. 3 D. 4
- 17. Suppose that a directed graph is to be stored and then queries for the presence of various edges will be submitted. Which of the following worst-case time bounds for testing whether one edge is present is **incorrect**? (Vertices are conveniently labeled by numbers $0, 1, \ldots, V 1$.)
 - A. Adjacency lists (ordered): $\Theta(\log V)$
 - B. Adjacency lists (unordered): $\Theta(V)$
 - C. Adjacency matrix: $\Theta(1)$

A. 1

- D. Compressed adjacency lists (ordered): $\Theta(\log V)$
- 18. The main disadvantage of compressed adjacency lists is:
 - A. It is difficult to change the graph
 - B. They waste space
 - C. Directed graphs may not be represented
 - D. Undirected graphs may not be represented
- 19. Suppose there is exactly one path from vertex 7 to vertex 10 in a directed graph:
 - $7 \rightarrow 8 \rightarrow 3 \rightarrow 5 \rightarrow 10$. During the scan of which column will Warshall's algorithm record the presence of this path?
 - A. 3 B. 5 C. 7 D. 8
- 20. The birthday paradox is about?

A. The probability that a UTA student has the same birthday as President Spaniolo

- B. The probability that some pair of persons in a room have the same birthday
- C. The probability that a CSE 2320 student knows something about hashing and collisions
- D. The probability that all persons in a room have the same birthday
- 21. The load factor, α , for hashing implemented with chaining when the number of linked lists is *m* and the number of stored records is *n* is defined as:

A.
$$\frac{m}{n}$$
 B. $\frac{n}{m}$ C. $\frac{1}{1-\alpha}$ D. $\frac{1}{n} \sum_{i=0}^{n-1} \frac{m}{m-i}$

22. Which of the following binary trees has exactly one legal coloring as a red-black tree?



Problems 23 and 24 refer to the following hash table whose keys are stored by linear probing using h(key) = key % 13.

0	1	2	3	4	5	6	7	8	9	10	11	12
			94		122	110	20	86	87	62		

- 23. 268 would be inserted into which slot of the given table? A. 0 B. 1 C. 2 D. 11
- 24. 169 would be inserted into which slot of the given table? (without previously inserting 268)
 - A. 0 B. 4 C. 6 D. 11
- 25. The cycle property for minimum spanning trees may be used to find an MST by:
 - A. Growing the MST by repeatedly including a minimum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
 - B. Growing the MST by repeatedly including a maximum weight edge from some vertex in the tree to some vertex that has not yet been placed in the tree.
 - C. Remove the minimum weight in any cycle until only a tree of edges remains.
 - D. Remove the maximum weight in any cycle until only a tree of edges remains.

Long Answer

 What are the entries in the heap (for Prim's algorithm) before and after moving the next <u>TWO</u> vertices and edges into the minimum spanning tree? DO NOT COMPLETE THE ENTIRE MST!!! Edges already in the MST are the thick ones. 10 points.



Perform depth-first search on the following graph, including start/finish times and edge types (T=tree, B=back, C=cross, F=forward.) Assume that the adjacency lists are <u>ordered</u>. Write your answer in the tables below. 10 points



3. Demonstrate the Floyd-Warshall algorithm, *with successors*, for the following graph. The paths indicated in the final matrix must have *at least one* edge. You *are not* required to show the intermediate matrices. 10 points.



4. Insert 65 into the given red-black tree. Be sure to indicate the cases that you used. (10 points)



5. Consider the following hash table whose keys were stored by double hashing using

 $h_1(\text{key}) = \text{key } \% \ 13 \text{ and } h_2(\text{key}) = 1 + (\text{key } \% \ 12).$

a. Give the number of probes needed to find each of the seven stored keys (using double hashing).
 (7 points)

120	186	187	162
122	110	194	

- b. Suppose 135 is to be inserted (using double hashing). Which slot will be used? (3 points)
- 6. (Extra credit) Give augmenting paths for determining a maximum flow and give a minimum cut for the following network. s is the source and t is the sink. (10 points)



Minimum Cut:

S vertices: s

T vertices: t

Augmenting Paths and Contribution to Flow: