Answer for Exercise 1.3 :

Dynamic list accessing problem :

1) ACCESS : The proof is the same as static list model

2) INSERTION : (To the end of list which has length $\ell-1$ before)

$$\hat{c}(MTF) = \ell - 1 + 1 = \ell$$

$$c(OPT) = \ell - F + P$$

$$\hat{c}(MTF) \leq c(OPT) \leq 2 \cdot c(OPT) - 1 \quad (c(OPT) - 1 \geq 0)$$

3) DELETION :

$$\hat{c}(MTF) = K - |WE| - |EW| \leq j - |EW| \leq j$$

$$c(OPT) = j + P = j + (P+1) - 1$$

So total cost $\leq 2j - 1$ since potential will pick up $P+1$ cost

Therefore $MTF(\sigma) \leq 2\,OPT(\sigma) - n$ with $n = |\sigma|$.

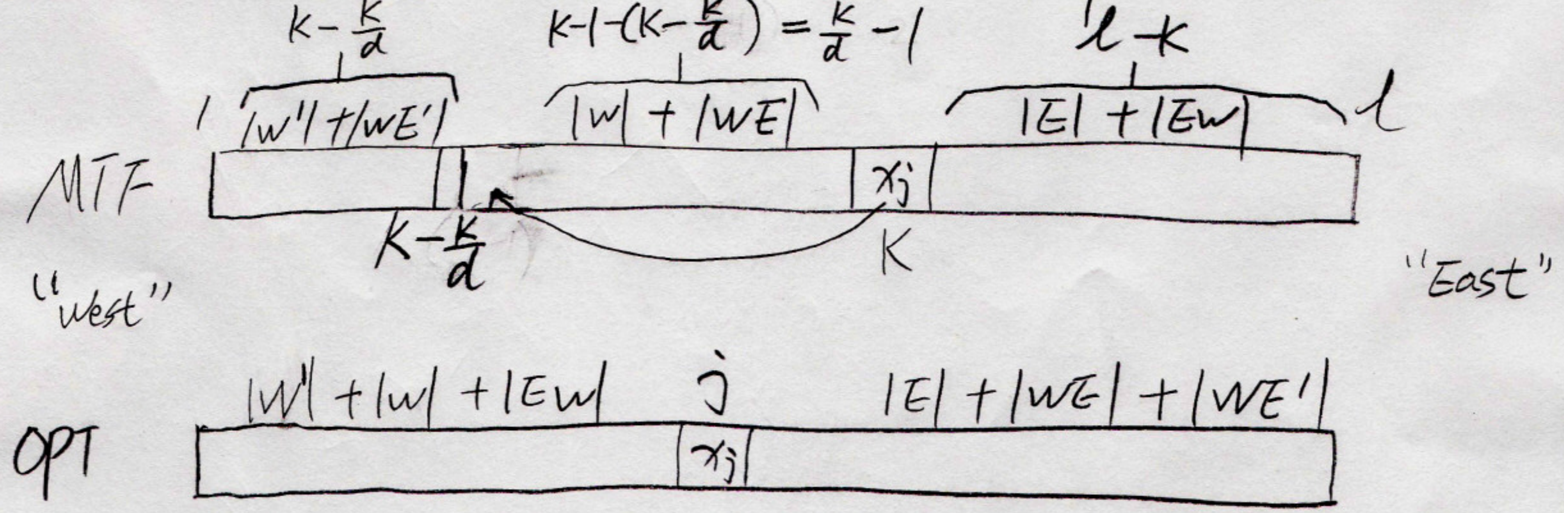
clearly $OPT(\sigma) \leq n\ell$, Hence, $\frac{OPT(\sigma)}{\ell} \leq n$, Therefore,

$$2\,OPT(\sigma) - n \leq 2\,OPT(\sigma) - \frac{OPT(\sigma)}{\ell}$$

$$\Rightarrow 2\,OPT(\sigma) - n \leq \left(2 - \frac{1}{\ell}\right) OPT(\sigma),$$

So $\left(2 - \frac{1}{\ell}\right)$ upper bound also holds for the dynamic list accessing problem where ℓ is the maximum number of items present in the list.

Answer for Exercise 1.4: I use k instead of i



W = Items "west" of x_j in both lists. In MTF, W refer to the items "west" of x_j but "East" of the item at position $k-\frac{k}{d}$ only

W' = Items "west" of x_j in both lists. In MTF, W' refer to the items "west" of the item at position $k-\frac{k}{d}$ only (include it)

WE = Items "west" of x_j and "east" of the item at position $k-\frac{k}{d}$ in MTF, but east of x_j in OPT

WE' = Items "west" of x_j and "west" of the item at position $k-\frac{k}{d}$ (include it) in MTF, but east of x_j in OPT

E = Items "east" of x_j in both lists

EW = Items east of x_j in MTF, but west of x_j in OPT.

(PP2)

Observations :

① $j = |W'| + |W| + |EW| + 1$

② $K = |W'| + |WE'| + |W| + |WE| + 1$

From ① we can get ③ : $j - 1 \geq |W|$ since $|EW| + |W'| \geq 0$

From ② and $|W'| + |WE'| = K - \frac{K}{d}$, we can get :

$$K = K - \frac{K}{d} + |W| + |WE| + 1$$

$$\Rightarrow K = d|W| + d|WE| + d$$

$$\Rightarrow d|WE| = K - d|W| - d \quad ④$$

Let potential function $\Phi = d *$ the number of inversions,

Amortized cost of ACCESS of x_j (by MTF):

search + $|W|$ new inversions $- |WE|$ lost inversions

$$= K + d|W| - d|WE|$$

$$\overset{④}{=} K + d|W| - K + d|W| + d$$

$$= 2d|W| + d = d(2|W| + 1)$$

$$\leq d(2(j-1) + 1) = d(2j - 1)$$

OPT actual cost $= j + dp$.

"correction" to Δ in $\Phi \leq -df$ for lost inversions $+ dp$ for new inversions

So after accessing and updating both lists, $\hat{C}_{MTF}$ is :

$K + d|W| - d|WE| - df + dp \leq d \cdot 2j - d + dp - df \leq d(2\,OPT - 1)$

The search cost s incurred by OPT for this request is exactly $\hat{j}$. when the request is an INSERT(x_j) or an ACCESS(x_j) when x_j is not on the list, exactly the same argument holds, with $\hat{j} = l+1$. In the case of deletion, the case is easier, no new inversions are being created and the contribution to the amortized cost is:

$$k - d|W\Xi| \leq d \cdot (2\hat{j} - 1) = d \cdot (2s - 1)$$

To show above, we need to show:

$$d|w| + d \leq d \cdot (2\hat{j} - 1) \quad (\text{since } k - d|W\Xi| \leq d|w| + d)$$

$$\Leftarrow |w| + 1 \leq (2\hat{j} - 1)$$

$$\Leftarrow |w| \leq \hat{j} - 1 \text{ and } 1 \leq \hat{j} \text{ which are true.}$$

Therefore, ACCESS(x_j), an INSERT(x_j), or a DELETE(x_j), is at most $d(2s - 1 + P - F)$

Because each free transposition contributes -1, each paid transposition contributes at most 1, since the -1s, one per operation, sum to $-n$. So

$$MTF(\sigma) \leq d \cdot (2 \cdot OPT_C(\sigma) + OPT_P(\sigma) - OPT_F(\sigma) - n)$$