CSE 5314
Computational Complexity

HW 2
Exercise 10.5

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Exercise 10.5

Given the conditions of Example 10.3, verify that $OPT(\sigma) = W_{\sigma}(ace) = 7$, where σ is the entire request sequence of that example

Conditions of Example 10.3

- 5-node weighted undirected complete graph with node set {a, b, c, d, e}.
- Edges between any two nodes in the subset {a, b, c, d} all have weight 1.
- All edges from nodes in the subset to e have weight 2

Dynamic programming (Lemma 10.2)

C: configuration

$W_\Phi(C)$: the configuration distance between C_0 and C

$W'(C) = W_{\sigma i r}(C)$ and $W(C) = W_{\sigma i}(C)$

$$\begin{cases} W_\Phi(C) = D(C_0, C) \\ W'(C) = \min\limits_{x \text{ is in } C} \{W(C - x + r) + d(r, x)\} \end{cases}$$

OPT cost

Given any initial configuration and any request sequence σ, the work function to compute the optimal offline cost to server σ:

$$OPT(\sigma) = \min W_{\sigma}(C)$$

Steps

1) Input element ‘e’:

   Suppose we start from the 2nd row, since e is in {a, c, e},

   then $W_{\sigma 1e}$ (ace) = W_{Φ}(ace) = 2

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
•	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3

Steps (cont.)

2) Input element ‘d’:

$W\sigma 2d(ace) = \min \{ W\sigma 1e(ace - x + d) + d(d, x)\}$

x is in {a,c,e}

$= \min \{ W\sigma 1e(cde) + d(d, a), W\sigma 1e(ade) + d(d, c), W\sigma 1e(acd) + d(d, e) \}$

$= \min \{3+1, 3+1, 4+2\} = 4$

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3

Steps (cont.)

3) Input element ‘a’:

Since a is in {a, c, e}, then clearly

$W_{\sigma 3a}$ (ace) = $W_{\sigma 2d}$ (ace) = 4

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4

Steps (cont.)

4) Input element ‘b’:

$W_{\sigma 4b}(ace) = \min_{x \text{ is in } \{a,c,e\}} \{ W_{\sigma 3a}(ace - x + b) + d(b, x)\}$

$= \min \{W_{\sigma 3a}(bce) + d(b, a), W_{\sigma 3a}(abe) + d(b, c), W_{\sigma 3a}(abc) + d(b, e) \}$

$= \min \{5+1, 4+1, 5+2\} = 5$

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5

Steps (cont.)

5) Input element ‘c’:

Since c is in {a, c, e}, then clearly $W_{\sigma 5c}(ace) = W_{\sigma 4b}(ace) = 5$

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5

Steps (cont.)

6) Input element ‘a’:

Since a is in {a, c, e}, then clearly $W_{\sigma 6a}(ace) = W_{\sigma 5c}(ace) = 5$

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6

Steps (cont.)

7) Input element ‘b’:

$W_{\sigma 7b}(ace) = \min_{x \text{ is in } \{a,c,e\}} \{ W_{\sigma 6a}(ace - x + b) + d(b, x)\}$

$= \min \{W_{\sigma 6a}(bce) + d(b, a), W_{\sigma 6a}(abe) + d(b, c), W_{\sigma 6a}(abc) + d(b, e) \}$

$= \min \{6+1, 6+1, 5+2\} = 7$

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6
b	5	6	6	6	7	7	6	6	7	7

Steps (cont.)

8) Input element ‘a’:

Since a is in {a, c, e}, then clearly $W_{\sigma 8a}$ (ace) = $W_{\sigma 7b}$ (ace) = 7

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6
b	5	6	6	6	7	7	6	6	7	7
					↓					
a	5	6	6	6	7	7	6	7	7	8

Steps (cont.)

9) Input element ‘c’:

Since c is in {a, c, e}, then clearly $W_{\sigma 9c}$ (ace) = $W_{\sigma 8a}$ (ace) = 7

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6
b	5	6	6	6	7	7	6	6	7	7
a	5	6	6	6	7	7	6	7	7	8
c	5	6	7	6	7	8	6	7	8	8

Steps (cont.)

10) Input element ‘e’:

Since e is in {a, c, e}, then clearly $W_{\sigma 10e}$ (ace) = $W_{\sigma 9c}$ (ace) = 7

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print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
*	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6
b	5	6	6	6	7	7	6	6	7	7
a	5	6	6	6	7	7	6	7	7	8
c	5	6	7	6	7	8	6	7	8	8
e	9	9	7	9	7	8	9	7	8	8

Review

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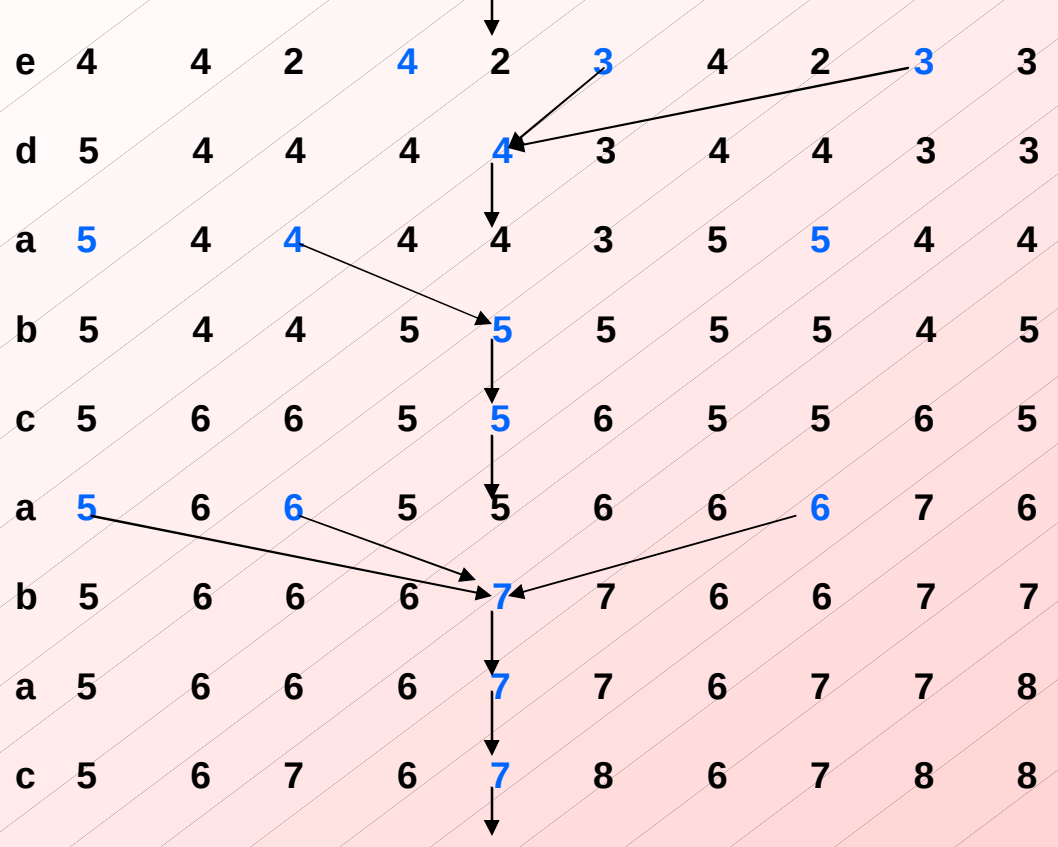
print updating results!

	abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
•	0	1	2	1	2	3	1	2	3	3
e	4	4	2	4	2	3	4	2	3	3
d	5	4	4	4	4	3	4	4	3	3
a	5	4	4	4	4	3	5	5	4	4
b	5	4	4	5	5	5	5	5	4	5
c	5	6	6	5	5	6	5	5	6	5
a	5	6	6	5	5	6	6	6	7	6
b	5	6	6	6	7	7	6	6	7	7
a	5	6	6	6	7	7	6	7	7	8
c	5	6	7	6	7	8	6	7	8	8
e	9	9	7	9	7	8	9	7	8	8



Questions?