Homework Set 1

Exercise 4.5

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Online Computation And Competitive Analysis

Borodin & El-Yaniv

Statement of Exercise

Prove that [*non-deterministic algorithm*] $\text{MARK}_{\text{RAND}}$ is H_k-competitive against an oblivious adversary when the total number of pages N of slow memory = $k + 1$.

Defend the Claim that:

$$\overline{\mathcal{R}}_{OBL}\left(MARK_{RAND}\right) \le H_k$$

Proof:

Previously established by proof in lecture:

$$\overline{\mathcal{R}}_{OBL}\left(MARK_{RAND}\right) \le 2H_k$$

This result was established with no restriction on the number of slow memory pages in excess of cache size (other obviously than being no less than k+1 to avoid the trivial situation of no faults by anybody and a ratio of one).

Proof:

Key step in that proof was establishment of the claim about ***m*** new-page requests in any *k*-phase partition *i and the total thereof for the entire request sequence*:

$$MARK_{RAND}(\sigma) \leq \left(\sum_{i=1}^{n} m_i \right) H_k$$

Proof:

If we are constrained to a slow memory exactly one memory page larger than size of cache, we are guaranteed in every *k*-phase partition, there can be no more than exactly one request for a new page not currently in cache, namely a request for that one slow memory page NOT in cache:

$$[N = k+1] \rightarrow (\forall i)[m_i \not> 1]$$

Proof:

If each of the n k-phase partitions contains no more than one request for a page not in cache, then all n k-phases of the sequence collectively cannot contain more than n such requests:

$$(\forall i)[m_i \not> 1] \rightarrow \left[\left(\sum_{i=1}^{n} m_i\right) \not> \left(\sum_{i=1}^{n} 1_i\right) = n\right]$$

$$\rightarrow \left[\left(\sum_{i=1}^{n} m_i\right) \le n\right]$$

Proof:

$$\left[\left(\sum_{i=1}^{n} m_i\right) \le n\right] \rightarrow \left[\left(\sum_{i=1}^{n} m_i\right) H_k \le nH_k\right]$$

$$\rightarrow \left[MARK_{RAND}(\sigma) \le \left(\sum_{i=1}^{n} m_i\right) H_k \le nH_k\right]$$

Also previously established in proof in lecture was the lower bound on the faults of an optimal algorithm regardless of cache size in the face of an oblivious adversary's request sequence:

$$n \le OPT(\sigma)$$

Proof:

If OPT's performance is independent of cache size, it is likewise independent of a slow memory size that is a function of cache size, hence, also in this situation:

$$n \le OPT(\sigma)$$

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Proof:

Therefore:

$$\left[MARK_{RAND}(\sigma) \le nH_k\right] \wedge \left[n \le OPT(\sigma)\right]$$

$$\rightarrow \left[MARK_{RAND}(\sigma) \le H_k \cdot OPT(\sigma)\right]$$

$$\rightarrow \left[\frac{MARK_{RAND}(\sigma)}{OPT(\sigma)} \le H_k\right]$$

$$\rightarrow \left[\overline{\mathcal{R}}_{OBL}\left(MARK_{RAND}\right) \le H_k\right] Q.E.D.$$

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