

# To Divide the Rent, Start With a Triangle: An old mathematical notion ...

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## To Divide the Rent, Start With a Triangle

An old mathematical notion can be used to fairly resolve thorny modern predicaments.

By ALBERT SUN

Last year, two friends and I moved into a small three-bedroom apartment in Manhattan. We chose it for its relatively reasonable price — around \$3,000 a month — and its convenient location. Just finding it was a challenge, but then we faced another one: deciding who would get each bedroom.

The bedrooms were different sizes, ranging from small to very small. Two faced north toward the street and had light; the third and smallest faced an alley. The largest had two windows; the midsize room opened onto the fire escape.

Every month, unrelated people move into apartments together to save on rent. Many decide to simply divide the rent evenly, or to base it on bedrooms' square footage or perhaps even on each resident's income.

But as it turns out, a field of academics is dedicated to studying the subject of fair division, or how to divide good and bad things fairly among groups of people. To the researchers, none of the typical methods are satisfactory. They have better ways.

The problem is that individuals evaluate a room differently. I care a lot about natural light, but not everyone does. Is it worth not having a closet? Or one might care more about the shape of the room, or its proximity to the bathroom.

A division of rent based on square feet or any fixed list of elements can't take every individual preference into account. And negotiation without a method may lead to conflict and resentment.

I set out to find a better way to divide our rent. That's how I came across a paper by Francis Su, a math professor at Harvey Mudd College in California, about a mathematical proposition discovered in 1923 by the German mathematician Emanuel Sperner. It is called Sperner's lemma.

The connection between Sperner's lemma and rent division was first published by Dr. Su in a 1999 paper titled "Rental Harmony: Sperner's Lemma in Fair Division." He came to the problem while completing his doctorate at Harvard. A friend of his was facing the same predicament I was —

### Rental Harmony Through Math

A mathematical theorem called Sperner's lemma can be used to divide unequal assets fairly.



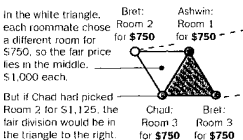
**THE PROBLEM** These hypothetical roommates, Ashwin, Bret and Chad, want to share an apartment. The total rent is \$3,000, but the rooms, here numbered 1 through 3, are all different. How can they choose rooms and divide the rent fairly?

#### THE SOLUTION

The triangle at right represents every possible combination of room prices. Each smaller triangle is a set of choices made by the three roommates.

#### SPERNER'S LEMMA

Sperner's Lemma guarantees that there is a small triangle where every roommate has picked a different room: The "fair" price lies somewhere between the prices of those three corners.



moving into a shared apartment with rooms of different sizes and features — and had asked for his advice.

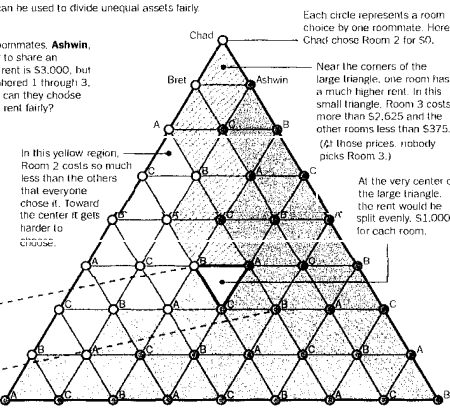
Dr. Su realized that it might be related to another problem he had heard about, in which a group had to divide a theoretical cake when some want frosted flowers or an edge with more frosting.

"The trick is to design a procedure to have everyone act in their own self-interest and have an outcome that's fair," he said in an interview.

Those working on fair division like to joke that it traces back to Solomon and the baby, Steven Brams, a professor of political science at New York University and a pioneer of the field, says both the Bible and the Talmud have examples.

The procedures have been used to divide such things as Germany after World War II, deep-sea mining rights, and property after a divorce or death.

To promote the use of the new methods being invented, Ariel D. Procaccia, a computer science professor at Carnegie Mellon University, has been working on a website, Spliddit, to help people use these methods to fairly divide things like the order of



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In this yellow region, Room 2 costs so much less than the others that everyone chooses it. Toward the center it gets harder to choose.

In the white triangle, each roommate chose a different room for \$750, so the fair price lies in the middle. \$1,000 each.

But if Chad had picked Room 2 for \$1,125, the fair division would be in the triangle to the right.

Chad  
Bret  
Ashwin

Near the corners of the large triangle, one room has a much higher rent. In this small triangle, Room 3 costs more than \$2,625 and the other rooms less than \$375. (C) Those prices, nobody picks Room 3.)

At the very center of the large triangle, the rent would be split evenly, \$1,000 for each room.

Each circle represents a room choice by one roommate. Here Chad chose Room 2 for \$0.

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names of co-authors on a scientific paper or prized possessions in a divorce.

"There are all these examples of really nice ways to solve the problem," he said, "but nobody's using them."

Sperner's lemma can help us find a fully labeled triangle, but how does that divide rent? Building on the work of two other mathematicians, Forest Simmons and Michael Starbird, Dr. Su realized that the small, fully labeled triangle could represent the rooms and prices in a hypothetical apartment. Based on people's decisions to label the triangles at each interior corner, an algorithm could be used to follow a winding path through an infinite field of simplices — triangles extended into any number of dimensions — starting from the largest and traveling into its interior in search of a point on the inside where everybody would choose a different room.

Fortunately, my housemates and I didn't have to play the game infinitely. To find a solution accurate to the dollar requires only a finite number of steps.

And it works for any number of people, because Sperner's lemma applies to all simplices, not just to triangles.

What this method guarantees is that the solution is "envy free," as game theorists put it. No one will want to swap his price and price for someone else's.

Part of the beauty of the approach is that you don't have to come up with numbers yourself. All it requires is that at each step you pick which room you like best based on the prices assigned to each room at that moment. As the method proceeds, the prices get closer and the decisions become harder, but it contains no surprises. You're never stuck with a price that you haven't chosen.

After his paper on the method was published, Dr. Su worked with one of his students, Elisha Peterson, to create a calculator to promote fair division.

My friends were skeptical when I tried to explain why we should use this method, but they ultimately agreed. We walked around our empty apartment and then walked through the steps together and arrived at a happy solution.

I ended up in the room with the fire escape, paying the middle amount — about \$100 less than the larger room and \$100 more than the one facing the alley.

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FRANCIS SU  
"RENTAL HARMONY" PAPER  
AUTHOR

#### ONLINE. TRY THIS AT HOME

► An interactive tool lets readers use Sperner's lemma to calculate fair rent. [nytimes.com/science](http://nytimes.com/science)