# MULTI-RESOLUTION STATISTICAL ANALYSIS ON GRAPH STRUCTURED DATA IN NEUROIMAGING

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# Overview

- Multi-resolution
- Wavelets
- Wavelets on Graph
- Graph Data in Medical Imaging
- Applications

# What is Multi-resolution View?

- Simple zoom (in or out) of a function
- Scale space theory
- Gaussian / Laplacian Pyramid


Figure: Example of Multi-resolution view of an image. Top: Images in fine to coarse scales are shown from left to right, Bottom: Laplacian of Images

# Why Multi-resolution?

- Invariant Shape Descriptors (e.g., SIFT)
- Context Analysis (e.g., texture analysis)
- Edge Detection
- Compression

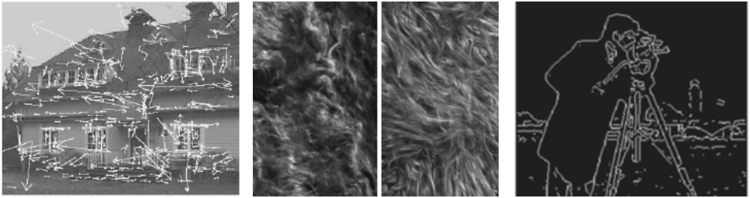
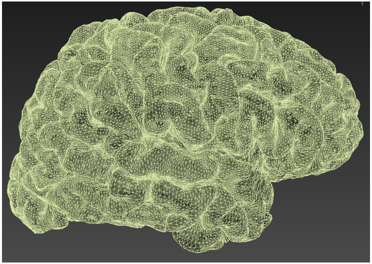


Figure: Left: SIFT features, Middle: Cancer vs Normal tissue, Right: Edge detection.

# Why Multi-resolution?

- Graph structured data in neuroimaging
- Vertices and Edges



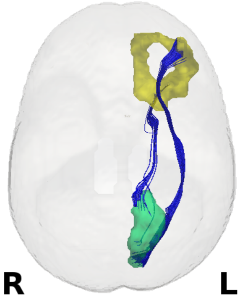
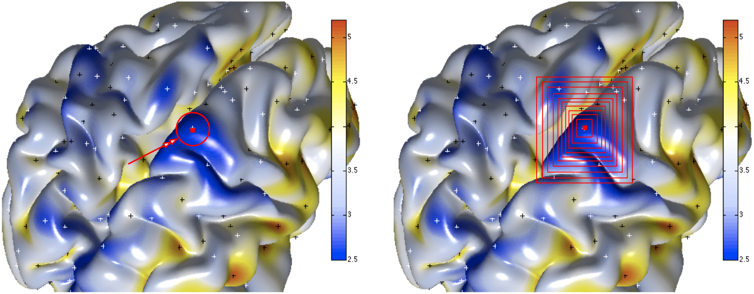



Figure: Left: cortical thickness, Right: neuron fiber between ROIs.

# Why Multi-resolution?

- Can we adopt Multi-resolution on **functions on Graphs**?
- Wavelet?



# Fourier Transform

- Fourier Series: Superposition of sinusoidal functions $e^{j\omega t}$
- Fourier Transform of $f(x)$ yields Fourier coefficients:

$$\hat{f}(\omega) = \int f(x) \mathrm{e}^{-j\omega x} \mathrm{d}x \tag{1}$$

- Inverse Fourier Transform reconstructs the original function:

$$f(x) = \frac{1}{2\pi} \int \hat{f}(\omega) \mathrm{e}^{j\omega x} \mathrm{d}\omega \tag{2}$$

# Fourier Basis vs. Wavelet Basis

- Fourier bases: Not localized in time, therefore causes artifacts.
- Wavelet bases: **Localized in both time and frequency**

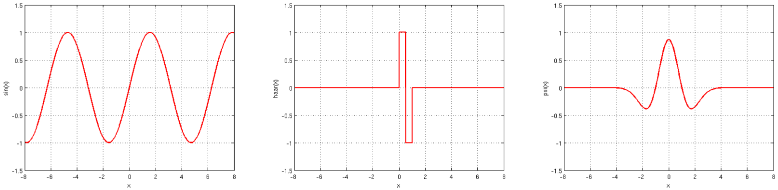


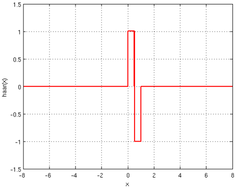

Figure: Left: Fourier basis, Middle: Haar Wavelet, Right: Mexican hat wavelet

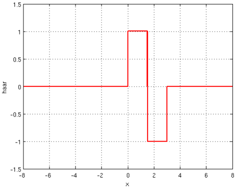
# Mother Wavelet

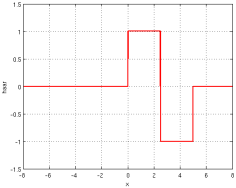
- Mother wavelet $\psi$:

$$\psi_{s,a}(x) = \frac{1}{s}\psi(\frac{x-a}{s}) \quad (3)$$

- Function with scale $s$ and translation $a$
- Scales (dilation $s$) of mother wavelet $\psi$:





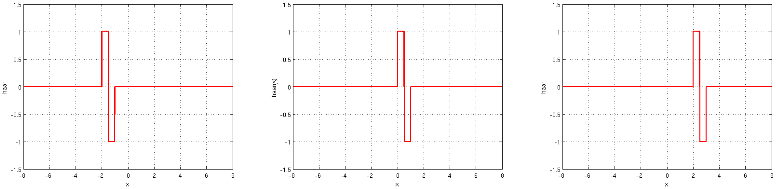


# Mother Wavelet

- Mother wavelet $\psi$:

$$\psi_{s,a}(x) = \frac{1}{s}\psi(\frac{x-a}{s}) \tag{4}$$

- Function with scale $s$ and translation $a$
- Translation (localization $a$) of mother wavelet $\psi$:



# Mother Wavelet in the Frequency Domain

- $\psi$ (blue) in the frequency domain: band-pass filters
- $\phi$ (red) in the frequency domain: low-pass filter

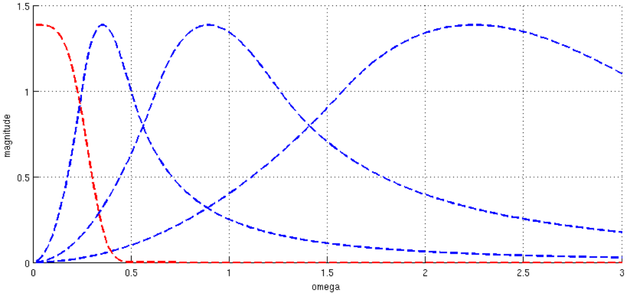



Figure: Example of a scaling function (red) and band-pass filters (blue) in the frequency domain.

# Continuous Wavelet Transform

- Wavelet Transform of $f(x)$:

$$W_f(s,a) = \langle f, \psi_{s,a} \rangle = \frac{1}{s} \int f(x) \psi^*(\frac{x-a}{s}) \mathrm{d}x \qquad (5)$$

- Outcome: wavelet coefficient $W_f(s,a)$

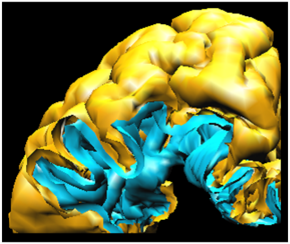
- Inverse wavelet transform (with $C_\psi = \int \frac{|\Psi(j\omega)|^2}{|\omega|} \mathrm{d}\omega < \infty$)

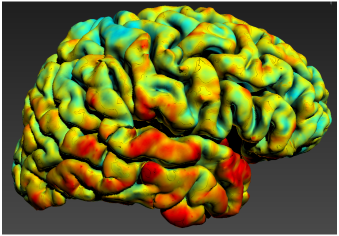
$$f(x) = \frac{1}{C_\psi} \iint W_f(s,a) \psi_{s,a}(x) \mathrm{d}a \, \mathrm{d}s \qquad (6)$$

- $\Psi(j\omega) = \int \psi(t) e^{-j\omega t} \mathrm{d}t$
- Outcome: reconstructed function $f(x)$

# Graph Structured Data in NeuroImaging

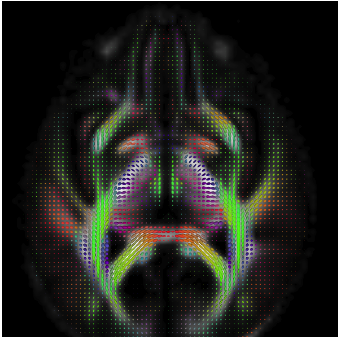
- Neuroimaging modalities with graph structures
  - Cortical thickness on a brain surface

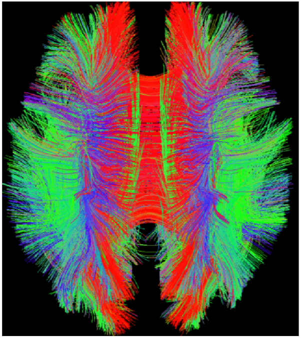




# Graph Structured Data in NeuroImaging

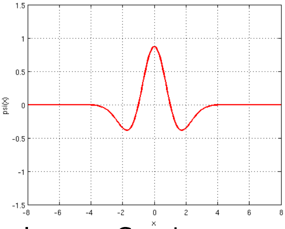
- Neuroimaging modalities with graph structures
  - Tractography using Diffusion Tensor Imaging (DTI)

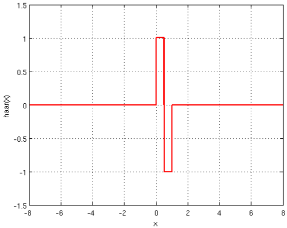




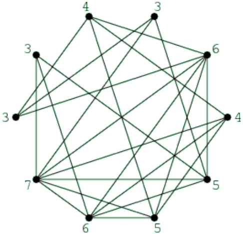
# Wavelet Transform on Graphs

- Wavelets in Euclidean Space



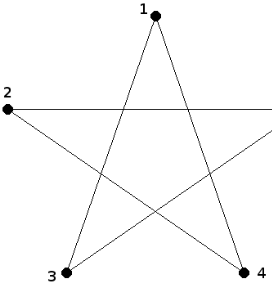




- Wavelets on Graphs
  - Scale? Translation?




# Wavelet Transform on Graphs

- Domain: $G = \{V, E, \omega\}$
  - $V$: vertex set, $E$: edge set, $\omega$: edge weight
- Construct filters in the frequency domain, and transform back to the original domain
- Ingredients: *Filters* and *Orthogonal Basis*



# Wavelet Transform on Graphs

- Spectral Graph Theory
  - Adjacency Matrix $A$: $a_{m,n}$ for connectivity information
  - Degree Matrix $D$: diagonal matrix with the sum of weights
  - Graph Laplacian: $L = D - A$
  - Eigenvector $\chi_l$ and eigenvalue $\lambda_l$ of $L$

$$0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{N-1} \tag{7}$$

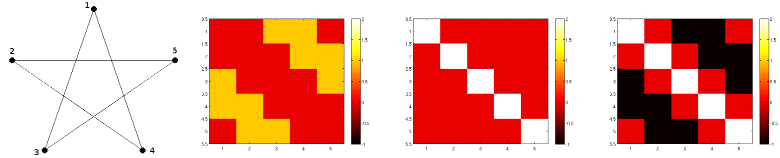



Figure: a) Star-shaped graph $G$, b) Adjacency matrix $A$ of $G$, c) Degree matrix $D$, d) Graph Laplacian $L$.

# Wavelet Transform on Graphs

- Graph Fourier transform of $f(n)$

$$\hat{f}(l) = \langle f, \chi_l \rangle = \sum_{n=1}^{N} f(n)\chi_l^*(n), \tag{8}$$

- Inverse graph Fourier transform

$$f(n) = \sum_{l=0}^{N-1} \hat{f}(l)\chi_l(n), \tag{9}$$

# Wavelet Transform on Graphs

- Define a kernel function $g$ – band-pass filters
- Wavelet function at node $m$, localized at node $n$ (with $\delta_n$)

$$\psi_{s,n}(m) = \sum_{l=0}^{N-1} g(s\lambda_l)\chi_l^*(n)\chi_l(m) \tag{10}$$

- Example of mother wavelets on a graph (surface mesh)

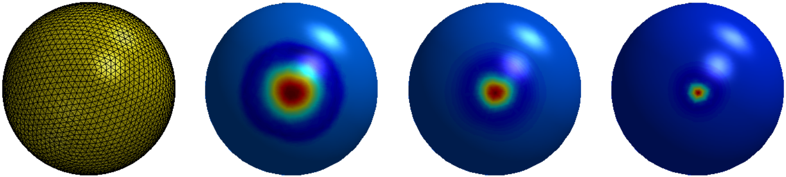


Figure: 3-D sphere mesh and Mexican hat wavelets in different scales localized at one vertex.

# Wavelet Transform on Graphs

- Forward and inverse wavelet transform of $f(n)$

$$W_f(s,n) = \sum_{l=0}^{N-1} g(s\lambda_l)\hat{f}(l)\chi_l(n) \tag{11}$$

$$f(n) = \frac{1}{C_g}\sum_{n=1}^{N}\int_0^\infty W_f(s,n)\psi_{s,n}(m)\frac{dt}{t} \tag{12}$$

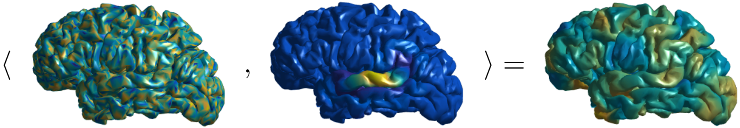


Figure: Example of wavelet basis on a brain surface, wavelet coefficients.

# Wavelet Transform on Graphs

- Forward and inverse wavelet transform of $f(n)$

$$W_f(s,n) = \sum_{l=0}^{N-1} g(s\lambda_l)\hat{f}(l)\chi_l(n) \tag{13}$$

$$f(n) = \frac{1}{C_g}\sum_{n=1}^{N}\int_0^\infty W_f(s,n)\psi_{s,n}(m)\frac{dt}{t} \tag{14}$$

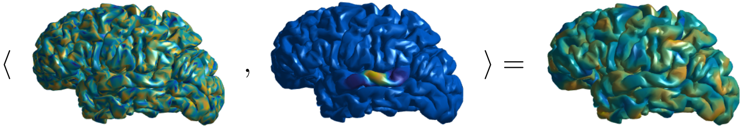


Figure: Example of wavelet basis on a brain surface, wavelet coefficients.

# Wavelet Transform on Graphs

- Forward and inverse wavelet transform of $f(n)$

$$W_f(s,n) = \sum_{l=0}^{N-1} g(s\lambda_l)\hat{f}(l)\chi_l(n) \tag{15}$$

$$f(n) = \frac{1}{C_g}\sum_{n=1}^{N}\int_0^\infty W_f(s,n)\psi_{s,n}(m)\frac{dt}{t} \tag{16}$$

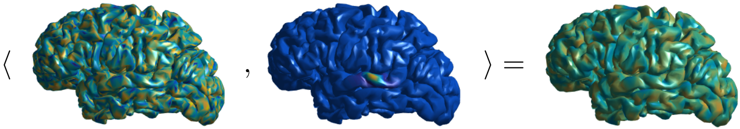


Figure: Example of wavelet basis on a brain surface and wavelet coefficients.

# Wavelet Transform on Graphs

- Forward and inverse wavelet transform of $f(n)$

$$W_f(s,n) = \sum_{l=0}^{N-1} g(s\lambda_l)\hat{f}(l)\chi_l(n) \tag{17}$$

$$f(n) = \frac{1}{C_g}\sum_{n=1}^{N}\int_0^\infty W_f(s,n)\psi_{s,n}(m)\frac{dt}{t} \tag{18}$$

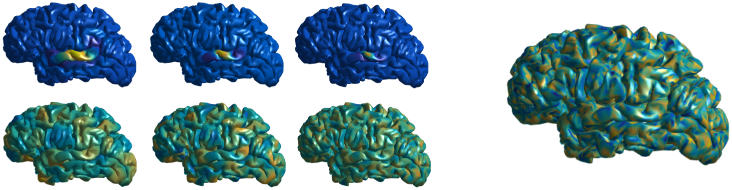


Figure: Inverse wavelet transform.

# Cortical Thickness Analysis

- Cortical thickness is the distance between inner and outer cortical surfaces.
- Data structure: cortical thickness values ($\sim 5mm$) on each vertex of a brain surface mesh ($\sim 160000$ vertices)

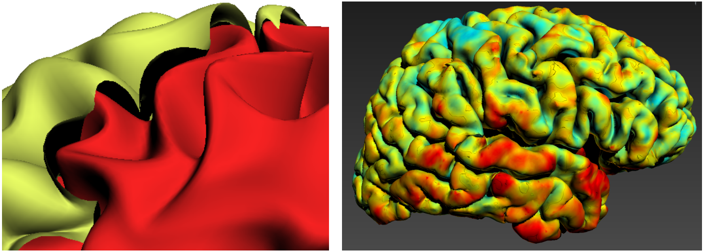


Figure: Cortical thickness on a brain surface. Left: brain surface mesh, Right: cortical thickness.

# Application: Cortical Signal Smoothing

- Cortical surface and thickness smoothing
- Incrementally add coarse to fine scale components

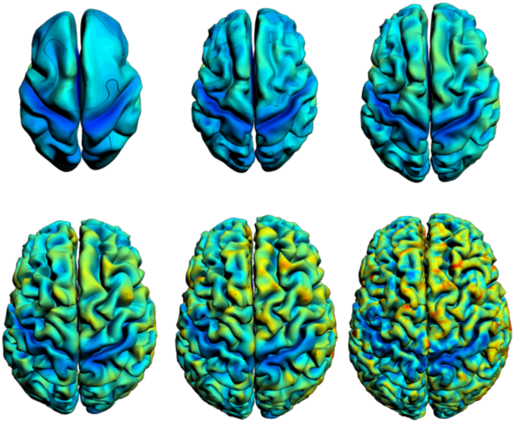


Figure: Cortical surface and thickness smoothing via wavelets on graphs

# Statistical Group Analysis

- Given: distributions of measurements from two groups (e.g., diseased vs. controls)
- Hypothesis testing by two sample $t$-test

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{vs.} \quad H_1 : \mu_1 - \mu_2 \neq 0$$

- Compute a test statistic and a $p$-value.
- Reject if $p$-value is under certain threshold (e.g., $0.05$ level)

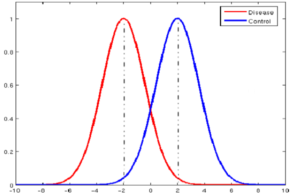


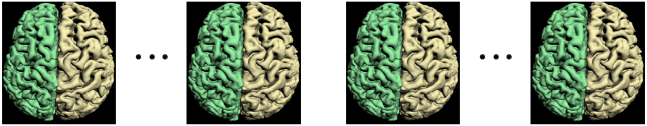

Figure: Distribution of signal measurements from two different groups.

# Application: Cortical Thickness Discrimination

- Given: measurements at each vertex
- Wavelet Multi-scale Descriptor (WMD): a set of wavelet coefficients at each vertex $n$

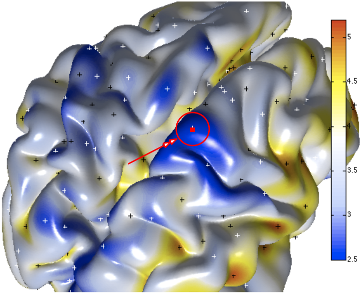
$$\mathrm{WMD}_f(n) = \{W_f(s,n) | s \in S\} \tag{19}$$

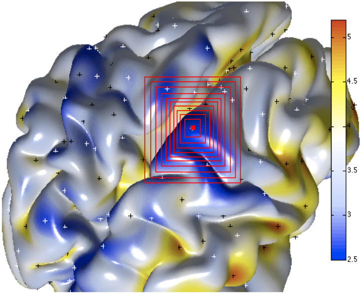
- Group analysis on AD vs. Control
- **Increase in sensitivity, decrease in sample sizes**



# Application: Cortical Thickness Discrimination

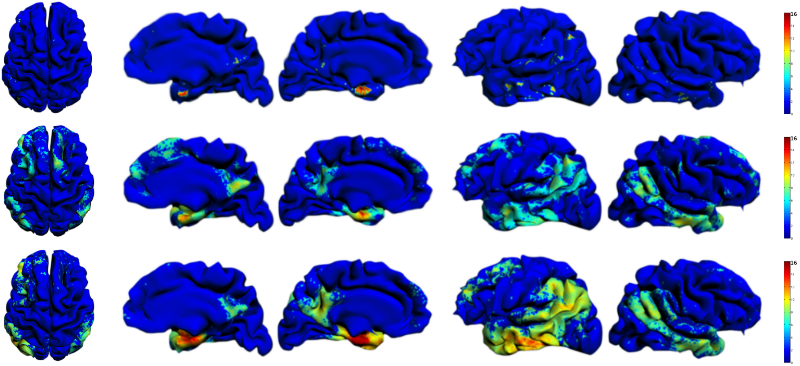
- Perform hypothesis testings at each vertex
- Project the resultant $p$-values on a template



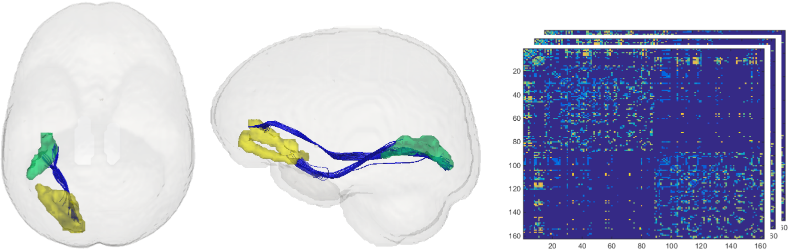
# Application: Cortical Thickness Discrimination

- ADNI dataset: 356 subjects (160 AD, 196 CN)
- Hotelling's $T^2$-test / False discovery rate (FDR)
- Precuneus, temporal/parietal regions, posterior cingulate, etc.



# Application: Brain Connectivity Discrimination

- Data structure: $162 \times 162$ adjacency matrix
  - Nodes: regions of interest (ROI)
  - Edges: 13401 connections with Fractional Anisotropy (FA)



# Application: Brain Connectivity Discrimination

- Analysis on functions on the edges, not on the vertices
- Requires transformation of the given data
- Line (dual) graph transform

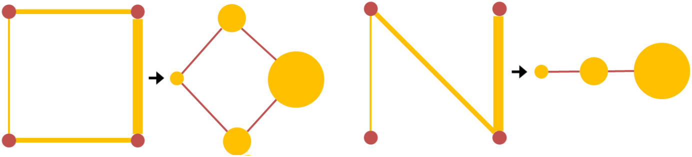


Figure: Examples of line graphs. Edges (weight: thickness) are represented as vertices (function: size) after transformation.

# Application: Brain Connectivity Discrimination

- ADRC dataset: 102 subjects (44 AD, 58 CN) with FA
- GLM controlling for age / gender and Bonferroni ($\alpha = 0.05$)
- Very few connections showing significant group difference

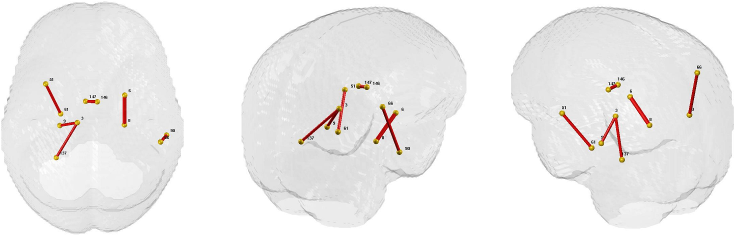


Figure: Significant group difference between AD and control groups. Color gives sign of strength: red (and blue) are stronger in controls (and AD group).

# Application: Brain Connectivity Discrimination

- ADRC dataset: 102 subjects (44 AD, 58 CN) with WMD
- MGLM controlling for age / gender and Bonferroni ($\alpha = 0.05$)
- Total of 81 connections showing significant group difference

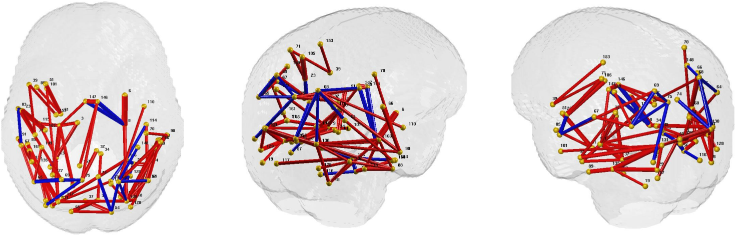


Figure: Significant group difference between AD and control groups. Color gives sign of strength: red (and blue) are stronger in controls (and AD group).

# Application: Brain Connectivity Discrimination

- Hub regions: ROI with connected edges $\geq 5$
  - Left superior and transverse occipital sulcus, Right hippocampus, Left superior parietal lobule, Right transverse occipital sulcus, Right precuneus, Right medial occipito-temporal gyrus.

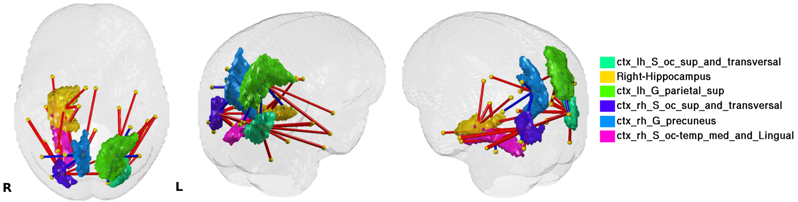



Figure: Illustration of the hub ROIs with connections identified as showing significant group difference between AD and control groups Color gives sign of strength: red (and blue) are stronger in controls (and AD group).

# Summary

- Multi-resolution
- Continuous wavelet transform
- Wavelet transform on graphs
- Application of wavelets in non-Euclidean space
  - Cortical thickness discrimination
  - Brain connectivity discrimination

# References


- Wavelets on Graphs via Spectral Graph Theory, Applied and Computational Harmonic Analysis, 2011
- Multi-resolutional shape features via non-Euclidean wavelets: Applications to statistical analysis of cortical thickness, NeuroImage, 2014
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